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THE LOGIC OF RELATIVES.

§ 1. *Three Grades of Clearness.*—The third volume of Professor Schröder's *Exact Logic*,[1] which volume bears separately the title I have chosen for this paper, is exciting some interest even in this country. There are in America a few inquirers into logic, sincere and diligent, who are not of the genus that buries its head in the sand,—men who devote their thoughts to the study with a view to learning something that they do not yet know, and not for the sake of upholding orthodoxy, or any other foregone conclusion. For them this article is written as a kind of popular exposition of the work that is now being done in the field of logic. To them I desire to convey some idea of what the new logic is, how two "algebras," that is, systems of diagrammatical representation by means of letters and other characters, more or less analogous to those of the algebra of arithmetic, have been invented for the study of the logic of relatives, and how Schröder uses one of these (with some aid from the other and from other notations) to solve some interesting problems of reasoning. I also wish to illustrate one other of several important uses to which the new logic may be put. To this end I must first clearly show what a relation is.

Now there are three grades of clearness in our apprehensions of the meanings of words. The first consists in the connexion of

[1] *Algebra und Logik der Relative.* Leipsic: B. G. Teubner. 1895. Price, 16 M.

the word with familiar experience. In that sense, we all have a clear idea of what *reality* is and what *force* is,—even those who talk so glibly of mental force being correlated with the physical forces. The second grade consists in the abstract definition, depending upon an analysis of just what it is that makes the word applicable. An example of defective apprehension in this grade is Professor Tait's holding (in an appendix to the reprint of his Britannica article, *Mechanics*) that energy is "objective" (meaning it is a substance), because it is permanent, or "persistent." For independence of time does not of itself suffice to make a substance; it is also requisite that the aggregant parts should always preserve their identity, which is not the case in the transformations of energy. The third grade of clearness consists in such a representation of the idea that fruitful reasoning can be made to turn upon it, and that it can be applied to the resolution of difficult practical problems.

§ 2. *Of the term Relation in its first Grade of Clearness.*—An essential part of speech, the Preposition, exists for the purpose of expressing relations. Essential it is, in that no language can exist without prepositions, either as separate words placed before or after their objects, as case-declensions, as syntactical arrangements of words, or some equivalent forms. Such words as "brother," "slayer," "at the time," "alongside," "not," "characteristic property" are relational words, or *relatives*, in this sense, that each of them *becomes a general name when another general name is affixed to it as object.* In the Indo-European languages, in Greek, for example, the so-called genitive case (an inapt phrase like most of the terminology of grammar) is, very roughly speaking, the form most proper to the attached name. By such attachments, we get such names as "brother *of* Napoleon," "slayer *of* giants," "*ἐπὶ Ἐλλισσαίου*, at the time of Elias," "*παρὰ ἀλλήλων*, alongside *of* each other," "not guilty," "a characteristic property *of* gallium." *Not* is a relative because it means "other than"; *scarcely*, though a relational word of highly complex meaning, is not a relative. It has, however, to be treated in the logic of relatives. Other relatives do not become general names until two or more names have been thus

affixed. Thus, "giver to the city" is just such a relative as the preceding; for "giver to the city of a statue of himself" is a complete general name (that is, there might be several such humble admirers of themselves, though there be but one, as yet); but "giver" requires *two* names to be attached to it, before it becomes a complete name. The dative case is a somewhat usual form for the second object. The archaic instrumental and locative cases were serviceable for third and fourth objects.

Our European languages are peculiar in their marked differentiation of common nouns from verbs. *Proper* nouns must exist in all languages; and so must such "pronouns," or indicative words, as *this*, *that*, *something*, *anything*. But it is probably true that in the great majority of the tongues of men, distinctive common nouns either do not exist or are exceptional formations. In their meaning as they stand in sentences, and in many comparatively widely-studied languages, common nouns are akin to participles, as being mere inflexions of verbs. If a language has a verb meaning "is a man," a noun "man" becomes a superfluity. For all men are mortals is perfectly expressed by "Anything either is-a-man not or is-a-mortal." Some man is a miser is expressed by "Something both is-a-man and is-a-miser." The best treatment of the logic of relatives, as I contend, will dispense altogether with class names and only use such verbs. A verb requiring an object or objects to complete the sense may be called a *complete relative*.

A verb by itself signifies a mere dream, an imagination unattached to any particular occasion. It calls up in the mind an *icon*. A *relative* is just that, an icon, or image, without attachments to experience, without "a local habitation and a name," but with indications of the need of such attachments.

An indexical word, such as a proper noun or demonstrative or selective pronoun, has force to draw the attention of the listener to some hecceity common to the experience of speaker and listener. By a hecceity, I mean, some element of existence which, not merely by the likeness between its different apparitions, but by an inward force of identity, manifesting itself in the continuity of its apparition throughout time and in space, is distinct from every-

thing else, and is thus fit (as it can in no other way be) to receive a proper name or to be indicated as *this* or *that*. Contrast this with the signification of the verb, which is sometimes in my thought, sometimes in yours, and which has no other identity than the agreement between its several manifestations. That is what we call an abstraction or idea. The nominalists say it is a *mere* name. Strike out the "mere," and this opinion is approximately true. The realists say it *is* real. Substitute for "is," *may be*, that is, *is* provided experience and reason shall, as their final upshot, uphold the truth of the particular predicate, and the natural existence of the law it expresses, and this is likewise true. It is certainly a great mistake to look upon an idea, merely because it has not the mode of existence of a hecceity, as a lifeless thing.

The proposition, or sentence, signifies that an eternal fitness, or truth, a permanent conditional force, or law, attaches certain hecceities to certain parts of an idea. Thus, take the idea of "buying by—of—from—in exchange for—." This has four places where hecceities, denoted by indexical words, may be attached. The proposition "A buys B from C at the price D," signifies an eternal, irrefragable, conditional force gradually compelling those attachments in the opinions of inquiring minds.

Whether or not there be in the reality any definite separation between the hecceity-element and the idea-element is a question of metaphysics, not of logic. But it is certain that in the expression of a fact we have a considerable range of choice as to how much we will denote by the indexical and how much signify by iconic words. Thus, we have stated "all men are mortal" in such a form that there is but one index. But we may also state it thus: "Taking anything, either it possesses not humanity or it possesses mortality." Here "humanity" and "mortality" are really proper names, or purely denotative signs, of familiar ideas. Accordingly, as here stated, there are three indices. Mathematical reasoning largely depends on this treatment of ideas as things; for it aids in the iconic representation of the whole fact. Yet for some purposes it is disadvantageous. These truths will find illustration in § 13 below.

Any portion of a proposition expressing ideas but requiring something to be attached to it in order to complete the sense, is in a general way relational. But it is only a *relative* in case the attachment of indexical signs will suffice to make it a proposition, or, at least, a complete general name. Such a word as *exceedingly* or *previously* is relational, but is not a relative, because significant words require to be added to it to make complete sense.

§ 3. *Of Relation in the Second Grade of Clearness.*—Is relation anything more than a connexion between two things? For example, can we not state that A gives B to C without using any other relational phrase than that one thing is connected with another? Let us try. We have the general idea of *giving*. Connected with it are the general ideas of *giver*, *gift*, and "*donee*." We have also a particular transaction connected with no general idea except through that of giving. We have a first party connected with this transaction and also with the general idea of giver. We have a second party connected with that transaction, and also with the general idea of "donee." We have a subject connected with that transaction and also with the general idea of gift. A is the only hecceity directly connected with the first party; C is the only hecceity directly connected with the second party, B is the only hecceity directly connected with the subject. Does not this long statement amount to this, that A gives B to C?

In order to have a distinct conception of Relation, it is necessary not merely to answer this question but to comprehend the reason of the answer. I shall answer it in the negative. For, in the first place, if relation were nothing but connexion of two things, all things would be connected. For certainly, if we say that A is unconnected with B, that non-connexion is a relation between A and B. Besides, it is evident that any two things whatever make a pair. Everything, then, is equally related to everything else, if mere connexion be all there is in relation. But that which is equally and necessarily true of everything is no positive fact, at all. This would reduce relation, considered as simple connexion between two things, to nothing, unless we take refuge in saying that relation *in general* is indeed nothing, but that *modes* of relation are some-

thing. If, however, these different modes of relation are different modes of connexion, relation ceases to be simple bare connexion. Going back, however, to the example of the last paragraph, it will be pointed out that the peculiarity of the mode of connexion of A with the transaction consists in A's being in connexion with an element connected with the transaction, which element is connected with the peculiar general idea of a *giver*. It will, therefore, be said, by those who attempt to defend an affirmative answer to our question, that the peculiarity of a mode of connexion consists in this, that that connexion is indirect and takes place through something which is connected with a peculiar general idea. But I say that is no answer at all; for if all things are equally connected, nothing can be more connected with one idea than with another. This is unanswerable. Still, the affirmative side may modify their position somewhat. They may say, we grant that it is necessary to recognise that relation is something more than connexion; it is *positive* connexion. Granting that all things are connected, still all are not positively connected. The various modes of relationship are, then, explained as above. But to this I reply: you propose to make the peculiarity of the connexion of A with the transaction depend (no matter by what machinery) upon that connexion having a positive connexion with the idea of a giver. But "positive connexion" is not enough; the relation of the general idea is quite peculiar. In order that it may be characterised, it must, on your principles, be made indirect, taking place through something which is itself connected with a general idea. But this last connexion is again more than a mere general positive connexion. The same device must be resorted to, and so on *ad infinitum*. In short, you are guilty of a *circulus in definiendo*. You make the relation of any two things consist in their connexion being connected with a general idea. But that last connexion is, on your own principles, itself a *relation*, and you are thus defining relation by relation; and if for the second occurrence you substitute the definition, you have to repeat the substitution *ad infinitum*.

The affirmative position has consequently again to be modified. But, instead of further tracing possible tergiversations, let us di-

rectly establish one or two positive positions. In the first place, I say that every relationship concerns some definite number of correlates. Some relations have such properties that this fact is concealed. Thus, any number of men may be brothers. Still, brotherhood is a relation between pairs. If A, B, and C are all brothers, this is merely the consequence of the three relations, A is brother of B, B is brother of C, C is brother of A. Try to construct a relation which shall exist either between two or between three things such as "—is either a brother or betrayer of—to—." You can only make sense of it by somehow interpreting the dual relation as a triple one. We may express this as saying that every relation has a definite number of blanks to be filled by indices, or otherwise. In the case of the majority of relatives, these blanks are qualitatively different from one another. These qualities are thereby communicated to the connexions.

In a complete proposition there are no blanks. It may be called a *medad*, or *medadic relative*, from *μηδαμός*, none, and *-άδα* the accusative ending of such words as *μόνας*, *δυάς*, *τριάς*, *τετράς*, etc.[1] A non-relative name with a substantive verb, as "—is a man," or "man that is—," or "—'s manhood" has one blank; it is a *monad*, or *monadic relative*. An ordinary relative with an active verb as "—is a lover of—" or "the loving by—of—" has two blanks; it is a *dyad*, or *dyadic relative*. A higher relative similarly treated has a plurality of blanks. It may be called a *polyad*. The rank of a relative among these may be called its *adinity*, that is, the peculiar quality of the number it embodies.

A *relative*, then, may be defined as the equivalent of a word or phrase which, either as it is (when I term it a *complete* relative), or else when the verb "is" is attached to it (and if it wants such attachment, I term it a *nominal* relative), becomes a sentence with some number of proper names left blank. A *relationship*, or *fundamentum relationis*, is a fact relative to a number of objects, consid-

[1] The Pythagoreans, who seem first to have used these words, probably attached a patronymic signification to the termination. A *triad* was derivative of *three*, etc.

ered apart from those objects, as if, after the statement of the fact, the designations of those objects had been erased. A *relation* is a relationship considered as something that may be said to be true of one of the objects, the others being separated from the relationship yet kept in view. Thus, for each relationship there are as many relations as there are blanks. For example, corresponding to the relationship which consists in one thing loving another there are two relations, that of loving and that of being loved by. There is a nominal relative for each of these relations, as "lover of—," and "loved by—." These nominal relatives belonging to one relationship, are in their relation to one another termed *correlatives*. In the case of a dyad, the two correlatives, and the corresponding relations are said, each to be the *converse* of the other. The objects whose designations fill the blanks of a complete relative are called the *correlates*. The correlate to which a nominal relative is attributed is called the *relate*.

In the statement of a relationship, the designations of the correlates ought to be considered as so many *logical subjects* and the relative itself as the *predicate*. The entire set of logical subjects may also be considered as a *collective subject*, of which the statement of the relationship is *predicate*.

§ 4. *Of Relation in the third Grade of Clearness.*—Mr. A. B. Kempe has published in the *Philosophical Transactions* a profound and masterly "Memoir on the Theory of Mathematical Form," which treats of the representation of relationships by "Graphs," which is Clifford's name for a diagram, consisting of spots and lines, in imitation of the chemical diagrams showing the constitution of compounds. Mr. Kempe seems to consider a relationship to be nothing but a complex of bare connexions of pairs of objects, the opinion refuted in the last section. Accordingly, while I have learned much from the study of his memoir, I am obliged to modify what I have found there so much that it will not be convenient to cite it; because long explanations of the relation of my views to his would become necessary if I did so.

A chemical atom is quite like a relative in having a definite number of loose ends or "unsaturated bonds," corresponding to

the blanks of the relative. In a chemical molecule, each loose end of one atom is joined to a loose end, which it is assumed must belong to some other atom, although in the vapor of mercury, in argon, etc., two loose ends of the same atom would seem to be joined; and why pronounce such hermaphrodism impossible? Thus the chemical molecule is a *medad*, like a complete proposition. Regarding proper names and other indices, after an "is" has been attached to them, as monads, they, together with other monads, correspond to the two series of chemical elements, H, Li, Na, K, Rb, Cs, etc., and Fl, Cl, Br, I. The dyadic relatives correspond to the two series, Mg, Ca, Sr, Ba, etc., and O, S, Se, Te, etc. The triadic relatives correspond to the two series B, Al, Zn, In, Tl, etc., and N, P, As, Sb, Bi, etc. Tetradic relatives are, as we shall see, a superfluity; they correspond to the series C, Si, Ti, Sn, Ta, etc. The proposition "John gives John to John" corresponds in

Fig. 1.

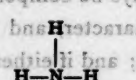

Fig. 2.

its constitution, as Figs. 1 and 2 show, precisely to ammonia.

But beyond this point the analogy ceases to be striking. In fact, the analogy with the ruling theory of chemical compounds quite breaks down. Yet I cannot resist the temptation to pursue it. After all, any analogy, however fanciful, which serves to focus attention upon matters which might otherwise escape observation is valuable. A chemical compound might be expected to be quite as much like a proposition as like an algebraical invariant; and the brooding upon chemical graphs has hatched out an important theory in invariants. Fifty years ago, when I was first studying chemistry, the theory was that every compound consisted of two oppositely electrified atoms or radicles; and in like manner every compound radicle consisted of two opposite atoms or radicles. The argument to this effect was that chemical attraction is evidently between things unlike one another and evidently has a saturation point; and further that we observe that it is the elements the most

extremely unlike which attract one another. Lothar Meyer's curve having for its ordinates the atomic volumes of the elements and for its abscissas their atomic weights tends to support the opinion that elements strongly to attract one another must have opposite characters; for we see that it is the elements on the steepest downward slopes of that curve which have the strongest attractions for the elements on the steepest upward inclines. But when chemists became convinced of the doctrine of valency, that is, that every element has a fixed number of loose ends, and when they consequently began to write graphs for compounds, it seems to have been assumed that this necessitated an abandonment of the position that atoms and radicles combine by opposition of characters, which had further been weakened by the refutation of some mistaken arguments in its favor. But if chemistry is of no aid to logic, logic here comes in to enlighten chemistry. For in logic, the medad must always be composed of one part having a negative, or antecedental, character, and another part of a positive, or consequental, character; and if either of these parts is compound its constituents are similarly related to one another. Yet this does not, at all, interfere with the doctrine that each relative has a definite number of blanks or loose ends. We shall find that, in logic, the negative character is a character of reversion in this sense, that if the negative part of a medad is compound, *its* negative part has, on the whole, a positive character. We shall also find, that if the negative part of a medad is compound, the bond joining its positive and negative parts has its character reversed, just as those relatives themselves have.

Several propositions are in this last paragraph stated about logical medads which now must be shown to be true. In the first place, although it be granted that every relative has a definite number of blanks, or loose ends, yet it would seem, at first sight, that there is no need of each of these joining no more than one other. For instance, taking the triad "—kills—to gratify—," why may not the three loose ends all join in one node and then be connected with the loose end of the monad "John is—" as in Fig. 3 making the proposition "John it is that kills what is John to gratify what

is John"? The answer is, that a little exercise of generalising power will show that such a four-way node is really a tetradic relative,

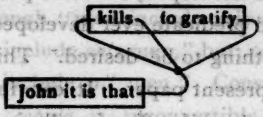

Fig. 3.

which may be expressed in words thus, "—is identical with—and with—and with—"; so that the medad is really equivalent to that

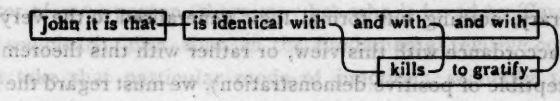

Fig. 4.

of Fig. 4, which corresponds to prussic acid as shown in Fig. 5.

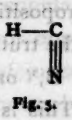

Fig. 5.

Thus, it becomes plain that every node of bonds is equivalent to a relative; and the doctrine of valency is established for us in logic. We have next to inquire into the proposition that in every combination of relatives there is a negative and a positive constituent. This is a corollary from the general logical doctrine of the illative character of the copula, a doctrine precisely opposed to the opinion of the quantification of the predicate. A satisfactory discussion of this fundamental question would require a whole article. I will only say in outline that it can be positively demonstrated in several ways that a proposition of the form "man = rational animal," is a compound of propositions each of a form which may be stated thus: "*Every* man (if there be any) is a rational animal" or "Men are *exclusively* (if anything) rational animals." Moreover, it must be acknowledged that the illative relation (that expressed by "therefore") is the most important of logical relations, the be-all and the end-all of the rest. It can be demonstrated that formal logic needs no other elementary logical relation than this;

but that with a symbol for this and symbols of relatives, including monads, and with a mode of representing the attachments of them, all syllogistic may be developed, far more perfectly than any advocate of the quantified predicate ever developed it, and in short in a way which leaves nothing to be desired. This in fact *will* be virtually shown in the present paper. It can further be shown that no other copula will of itself suffice for all purposes. Consequently, the copula of equality ought to be regarded as merely derivative.

Now, in studying the logic of relatives we must sedulously avoid the error of regarding it as a highly specialised doctrine. It is, on the contrary, nothing but formal logic generalised to the very tip-top. In accordance with this view, or rather with this theorem (for it is susceptible of positive demonstration), we must regard the *relative copula*, which is the bond between two blanks of relatives, as only a generalisation of the ordinary copula, and thus of the "*ergo*." When we say that from the proposition A the proposition B necessarily follows, we say that "the truth of A in *every way* in which it can exist at all is the truth of B," or otherwise stated "A is true *only* in so far as B is true." This is the very same relation which we express when we say that "*every* man is mortal," or "men are *exclusively* mortals." For this is the same as to say, "Take anything whatever, M; then, if M is a man, it follows necessarily that M is mortal." This mode of junction is essentially the same as that between the relatives in the compound relative "lover, in *every* way in which it may be a lover at all, of a servant," or, otherwise expressed, "lover (if at all) *exclusively* of servants." For to say that "Tom is a lover (if at all) only of servants of Dick," is the same as to say "Take anything whatever, M; then, if M is loved by Tom, M is a servant of Dick," or "everything there may be that is loved by Tom is a servant of Dick."

Now it is to be observed that the illative relation is not simply convertible; that is to say, that "from A necessarily follows B" does not necessarily imply that "from B necessarily follows A." Among the vagaries of some German logicians of some of the inexact schools, the convertibility of illation (like almost every other imaginable absurdity) has been maintained; but all the other in-

exact schools deny it, and exact logic condemns it, at once. Consequently, the copula of inclusion, which is but the *ergo* freed from the accident of asserting the truth of its antecedent, is equally inconvertible. For though "men include only mortals," it does not follow that "mortals include *only* men," but, on the contrary, what follows is "mortals include *all* men." Consequently, again, the fundamental *relative copula* is inconvertible. That is, because "Tom loves (if anybody) only a servant (or servants) of Dick," it does not follow that "Dick is served (if at all) only by somebody loved by Tom," but, on the contrary, what follows is "Dick is master of *every* person (there may be) who is loved by Tom." We thus see clearly, first, that, as the fundamental relative copula, we must take that particular mode of junction; secondly, that that mode is at bottom the mode of junction of the *ergo*, and so joins a relative of antecedental character to a relative of consequental character; and, thirdly, that that copula is inconvertible, so that the two kinds of constituents are of opposite characters. There are, no doubt, convertible modes of junction of relatives, as in "lover of a servant;"[1] but it will be shown below that these are complex and indirect in their constitution.

[1] Professor Schröder proposes to substitute the word "symmetry" for *convertibility*, and to speak of *simply convertible* modes of junction as "symmetrical." Such an example of wanton disregard of the admirable traditional terminology of logic, were it widely followed, would result in utter uncertainty as to what any

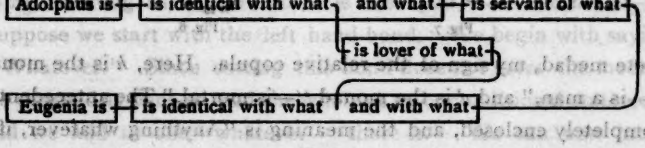

Fig. 6.

writer on logic might mean to say, and would thus be utterly fatal to all our efforts to render logic exact. Professor Schröder denies that the mode of junction in "lover of a servant" is "symmetrical," which word in practice he makes synonymous with "commutative," applying it only to such junctions as that between "lover" and "servant" in "Adolphus is at once lover and servant of Eugenia." Commutativity depends on one or more polyadic relatives having two like blanks as shown in Fig. 6.

It remains to be shown that the antecedent part of a medad has a negative, or reversed, character, and how this, in case it be compound, affects both its relatives and their bonds. But since this matter is best studied in examples, I will first explain how I propose to draw the logical graphs.

It is necessary to use, as the sign of the relative copula, some symbol which shall distinguish the antecedent from the consequent; and since, if the antecedent is compound (owing to the very character which I am about to demonstrate, namely, its reversing the characters of the relatives and the bonds it contains), it is very important to know just how much is included in that antecedent, while it is a matter of comparative indifference how much is included in the consequent (though it is simply everything not in the antecedent), and since further (for the same reason) it is important to know how many antecedents, each after the first a part of another, contain a given relative or copula, I find it best to make the line which joins antecedent and consequent encircle the whole of the former. Letters of the alphabet may be used as abbreviations of complete relatives; and the proper number of bonds may be attached to each. If one of these is encircled, that circle must have a bond corresponding to each bond of the encircled letter. Chemists sometimes write above atoms Roman numerals to indicate their *adinities;* but I do not think this necessary. Fig. 7 shows, in a com-

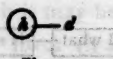

Fig. 7.

Fig. 8.

plete medad, my sign of the relative copula. Here, *h* is the monad "—is a man," and *d* is the monad "—is mortal." The antecedent is completely enclosed, and the meaning is "Anything whatever, if it be a man, is mortal." If the circle encloses a dyadic or polyadic relative, it must, of course, have a tail for every bond of that relative. Thus, in Fig. 8, *l* is the dyad "—loves—," and it is important to remark that the bond to the left is the lover and that to the right is the loved. Monads are the only relatives for which we need not be attentive to the positions of attachment of the bonds. In this figure,

w is the monad "—is wise," and *v* is the monad "—is virtuous." The *l* and *v* are enclosed in a large common circle. Had this not been done, the medad could not be read (as far as any rules yet given show), because it would not consist of antecedent and consequent. As it is, we begin the reading of the medad at the bond connecting antecedent and consequent. Every bond of a logical graph denotes a hecceity; and every unencircled bond (as this one is) stands for any hecceity the reader may choose from the universe. This medad evidently refers to the universe of men. Hence the interpretation begins: "Let M be any man you please." We proceed along this bond in the direction of the antecedent, and on entering the circle of the antecedent we say: "If M be." We then enter the inner circle. Now, entering a circle means a relation to *every*. Accordingly we add "whatever." Traversing *l* from left to right, we say "lover." (Had it been from right to left we should have read it "loved.") Leaving the circle is the mark of a relation "only to," which words we add. Coming to *v* we say "what is virtuous." Thus our antecedent reads: "Let M be any man you please. If M be whatever it may that is lover only to the virtuous." We now return to the consequent and read, "M is wise." Thus the whole means, "Whoever loves only the virtuous is wise."

As another example, take the graph of Fig. 9, where *l* has the

Fig. 9.

same meaning as before and *m* is the dyad "—is mother of—." Suppose we start with the left hand bond. We begin with saying "Whatever." Since cutting this bond does not sever the medad, we proceed at once to read the whole as an unconditional statement and we add to our "whatever" "there is." We can now move round the ring of the medad either clockwise or counter-clockwise. Taking the last way, we come to *l* from the left hand and therefore add "is a lover." Moving on, we enter the circle round *m*; and entering a circle is a sign that we must say "of *every thing* that." Since we pass through *m* backwards we do not read "is mother" but "is mothered" or "has for mother." Then, since we pass *out*

of the circle we should have to add "only"; but coming back, as we do, to the starting point, we need only say "that same thing." Thus, the interpretation is "Whatever there is, is lover of everything that has for mother that same thing," or "Every woman loves everything of which she is mother." Starting at the same point and going round the other way, the reading would be "Everybody is mother (if at all) only of what is loved by herself." Starting on the right and proceeding clockwise, "Everything is loved by every mother of itself." Proceeding counter-clockwise, "Everything has for mothers only lovers of itself."

Triple relatives afford no particular difficulty. Thus, in Fig. 10, w and v have the same significations as before; r is the monad, "—is a reward," and g is the triad "—gives — to —." It can be read either

Fig. 10.

"Whatever is wise gives every reward to every virtuous person," or "Every virtuous person has every reward given to him by everybody that is wise," or "Every reward is given by everybody who is wise to every virtuous person."

A few more examples will be instructive. Fig. 11, where A is the proper name Alexander means "Alexander loves only the virtuous," i. e., "Take anybody you please; then, if he be Alexander and if he loves anybody, this latter is virtuous."

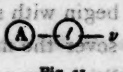

Fig. 11.

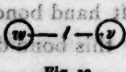

Fig. 12.

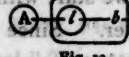

Fig. 13.

If you attempt, in reading this medad, to start to the right of l, you fall into difficulty, because your antecedent does not then consist of an antecedent and consequent, but of two circles joined by a bond, a combination to be considered below. But Fig. 12 may be read with equal ease on whichever side of l you begin, whether as "whoever is wise loves everybody that is virtuous," or "whoever is virtuous is loved by everybody that is wise." If in Fig. 13

$-b-$ be the dyad "—is a benefactor of—," the medad reads, "Alexander stands only to virtuous persons in the relation of loving only their benefactors."

Fig. 14, where $-s-$ is the dyad "—is a servant of —" may be read, according to the above principles, in the several ways following:

"Whoever stands to any person in the relation of lover to none but his servants benefits him."

"Every person stands only to a person benefited by him in the relation of a lover only of a servant of that person."

Fig. 14.

Fig. 15.

"Every person, M, is benefactor of everybody who stands to M in the relation of being served by everybody loved by him."

"Every person, N, is benefited by everybody who stands to N in the relation of loving only servants of him."

"Every person, N, stands only to a benefactor of N in the relation of being served by everybody loved by him."

"Take any two persons, M and N. If, then, N is served by every lover of M, N is benefited by M."

Fig. 15 represents a medad which means, "Every servant of any person, is a benefactor of whomever may be loved by that person." Equivalent statements easily read off from the graphs are as follows:

"Anybody, M, no matter who, is servant (if at all) only of somebody who loves (if at all) only persons benefited by M."

"Anybody, no matter who, stands to every master of him in the relation of benefactor of whatever person may be loved by him."

"Anybody, no matter who, stands to whoever loves him in the relation of being benefited by whatever servant he may have."

"Anybody, N, is loved (if at all) only by a person who is served (if at all) only by benefactors of N."

"Anybody, no matter who, loves (if at all) only persons benefited by all servants of his."

"Anybody, no matter who, is served (if at all) only by benefactors of everybody loved by him."

I will now give an example containing triadic relatives, but no monads. Let β be "—prevents—from communicating with—," the second blank being represented by a bond from the right of β and the third by a bond from below β. Let β mean "—would betray—to—," the arrangement of bonds being the same as with β. Then, Fig. 16 means that "whoever loves only persons who pre-

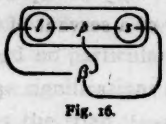

Fig. 16.

vent every servant of any person, A, from communicating with any person, B, would betray B to A." I will only notice one equivalent statement, viz.: "Take any three persons, A, B, C, no matter who. Then, either C betrays B to A, or else two persons, M and N, can be found, such that M does not prevent N from communicating with B, although M is loved by C and N is a servant of A."

This last interpretation is an example of the method which is, by far, the plainest and most unmistakable of any in complicated cases. The rule for producing it is as follows:

1. Assign a letter of the alphabet to denote the hecceity represented by each bond.[1]

2. Begin by saying: "Take any things you please, namely," and name the letters representing bonds not encircled; then add, "Then suitably select objects, namely," and name the letters representing bonds each once encircled; then add, "Then take any things you please, namely," and name the letters representing bonds each twice encircled. Proceed in this way until all the letters

[1] In my method of graphs, the spots represent the relatives, their bonds the hecceities; while in Mr. Kempe's method, the spots represent the objects, whether individuals or abstract ideas, while their bonds represent the relations. Hence, my own exclusive employment of bonds between pairs of spots does not, in the least, conflict with my argument that in Mr. Kempe's method such bonds are insufficient.

representing bonds have been named, no letter being named until all those encircled fewer times have been named; and each hecceity corresponding to a letter encircled odd times is to be suitably chosen according to the intent of the assertor of the medad proposition, while each hecceity corresponding to a bond encircled even times is to be taken as the interpreter or the opponent of the proposition pleases.

3. Declare that you are about to make statements concerning certain propositions, to which, for the sake of convenience, you will assign numbers in advance of enunciating them or stating their relations to one another. These numbers are to be formed in the following way. There is to be a number for each letter of the medad (that is for those which form spots of the graph, not for the letters assigned by clause 1 of this rule to the bonds), and also a number for each circle round more than one letter; and the first figure of that number is to be a 1 or a 2, according as the letter or the circle is in the principal antecedent or the principal consequent; the second figure is to be 1 or 2, according as the letter or the circle belongs to the antecedent or the consequent of the principal antecedent or consequent, and so on.

Declare that one or other of those propositions whose numbers contain no 1 before the last figure is true. Declare that each of those propositions whose numbers contain an odd number of 1's before the last figure consists in the assertion that *some one* or another of the propositions whose numbers commence with its number is true. For example, 11 consists in the assertion that either 111 or 1121 or 1122 is true, supposing that these are the only propositions whose numbers commence with 11. Declare that each of those propositions whose numbers contain an even number of 1's (or none) before the last figure consists in the assertion that *every one of* the propositions whose numbers commence with its number is true. Thus, 12 consists in the assertion that 121, 1221, 1222 are all true, provided those are the only propositions whose numbers commence with 12. The process described in this clause will be abridged except in excessively complicated cases.

4. Finally, you are to enunciate all those numbered proposi-

tions which correspond to single letters. Namely, each proposition whose number contains an even number of 1's, will consist in affirming the relative of the spot-letter to which that number corresponds after filling each blank with that bond-letter which by clause 1 of this rule was assigned to the bond at that blank. But if the number of the proposition contains an odd number of 1's, the relative, with its blanks filled in the same way, is to be denied.

In order to illustrate this rule, I will restate the meanings of the medads of Figs. 7–16, in all the formality of the rule; although such formality is uncalled for and awkward, except in far more complicated cases.

Fig. 7. Let A be anything you please. There are two propositions, 1 and 2, one of which is true. Proposition 1 is, that A is not a man. Proposition 2 is, that A is mortal. More simply, Whatever A may be, either A is not a man or A is mortal.

Fig. 8. Let A be anybody you please. Then, I will find a person, B, so that either proposition 1 or proposition 2 shall be true. Proposition 1 asserts that both propositions 11 and 12 are true. Proposition 11 is that A loves B. Proposition 12 is that B is not virtuous. Proposition 2 is that A is wise. More simply, Take anybody, A, you please. Then, either A is wise, or else a person, B, can be found such that B is not virtuous and A loves B.

Fig. 9. Let A and B be any persons you please. Then, either proposition 1 or proposition 2 is true. Proposition 1 is that A is not a mother of B. Proposition 2 is that A loves B. More simply, whatever two persons A and B may be, either A is not a mother of B or A loves B.

Fig. 10. Let A, B, C be any three things you please. Then, one of the propositions numbered, 1, 21, 221, 222 is true. Proposition 1 is that A is not wise. Proposition 21 is that B is not a reward. Proposition 221 is that C is not virtuous. Proposition 222 is that A gives B to C. More simply, take any three things, A, B, C, you please. Then, either A is not wise, or B is not a reward, or C is not virtuous, or A gives B to C.

Fig. 11. Take any two persons, A and B, you please. Then, one of the propositions 1, 21, 22 is true. 1 is that A is not Alex-

ander. 21 is that A does not love B. Proposition 3 is that B is virtuous.

Fig. 12. Take any two persons, A and B. Then, one of the propositions 1, 21, 22 is true. 1 is that A is not wise. 21 is that B is not virtuous. 22 is that A loves B.

Fig. 13. Take any two persons, A and C. Then a person, B can be found such that one of the propositions 1, 21, 22 is true. Proposition 21 asserts that both 211 and 212 are true. Proposition 1 that A is not Alexander. Proposition 211 is that A loves B. Proposition 212 is that B does not benefit C. Proposition 22 is that C is virtuous. More simply, taking any two persons, A and C, either A is not Alexander, or C is virtuous, or there is some person, B, who is loved by A without benefiting C.

Fig. 14. Take any two persons, A and B, and I will then select a person C. Either proposition 1 or proposition 2 is true. Proposition 1 is that both 11 and 12 are true. Proposition 11 is that A loves C. Proposition 12 is that C is not a servant of B. Proposition 2 is that A benefits B. More simply, of any two persons, A and B, either A benefits the other, B, or else there is a person, C, who is loved by A but is not a servant of B.

Fig. 15. Take any three persons, A, B, C. Then one of the propositions 1, 21, 22 is true. 1 is that A is not a servant of B; 21 is that B is not a lover of C; 22 is that A benefits C.

Fig. 16. Take any three persons, A, B, C. Then I can so select D and E, that one of the propositions 1 or 2 is true. 1 is that 11 and 121 and 122 are all true. 11 is that A loves D, 121 is that E is a servant of C, 122 is that D does not prevent E from communicating with B. 2 is that A betrays B to C.

I have preferred to give these examples rather than fill my pages with a dry abstract demonstration of the correctness of the rule. If the reader requires such a proof, he can easily construct it. This rule makes evident the reversing effect of the encirclements, not only upon the "quality" of the relatives as affirmative or negative, but also upon the selection of the hecceities as performable by advocate or opponent of the proposition, as well as upon the conjunctions of the propositions as disjunctive or conjunctive, or

(to avoid this absurd grammatical terminology) as alternative or simultaneous.

It is a curious example of the degree to which the thoughts of logicians have been tied down to the accidents of the particular language they happened to write (mostly Latin), that while they hold it for an axiom that two *nots* annul one another, it was left for me to say as late as 1867[1] that *some* in formal logic ought to be understood, and could be understood, so that *some-some* should mean *any*. I suppose that were ordinary speech of any authority as to the forms of logic, in the overwhelming majority of human tongues two negatives intensify one another. And it is plain that if "not" be conceived as less than anything, what is less than that is *a fortiori* not. On the other hand, although *some* is conceived in our languages as *more than none*, so that two "somes" intensify one another, yet what it ought to signify for the purposes of syllogistic is that, instead of the selection of the instance being left,—as it is, when we say "any man is not good,"—to the opponent of the proposition, when we say "some man is not good," this selection is transferred to the opponent's opponent, that is to the defender of the proposition. Repeat the some, and the selection goes to the opponent's opponent's opponent, that is, to the opponent again, and it becomes equivalent to *any*. In more formal statement, to say "Every man is mortal," or "Any man is mortal," is to say, "A man, as suitable as any to prove the proposition false, is mortal," while "Some man is mortal" is equivalent to "A man, as suitable as any to prove the proposition *not* false, is mortal." "Some-some man is mortal" is accordingly "A man, as suitable as any to prove the proposition *not not*-false, is mortal."

In like manner, encircled $2N+1$ times, a disjunctive conjunction of propositions becomes a copulative conjunction. Here, the case is altogether similar. Encircled even times, the statement is that some one (or more) of the propositions is true; encircled odd times, the statement is that any one of the propositions is true.

[1] "On the Natural Classification of Arguments." *Proceedings of the American Academy of Arts and Sciences.*

The negative of "lover of every servant" is "non-lover of some servant." The negative of "lover every way (that it is a lover) of a servant" is "lover some way of a non-servant."

The general nature of a relative and of a medad has now been made clear. At any rate, it will become so, if the reader carefully goes through with the explanations. We have not, however, as yet shown how every kind of proposition can be graphically expressed, nor under what conditions a medad is necessarily true. For that purpose it will be necessary to study certain special logical relatives.

§ 5. *Triads the primitive relatives.*—That out of triads all polyads can be constructed is made plain by Fig. 17.

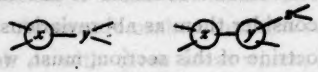

Fig. 17.

Fig. 18 shows that from two triads a dyad can be made. Fig. 19 shows that from one triad a monad can be made. Fig. 20 shows

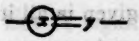

Fig. 18.

Fig. 19.

Fig. 20.

that from any even number of triads a medad can be made. In general, the union of a μ-ad and a ν-ad gives a $(\mu+\nu-2\lambda)$-ad, where λ is the number of bonds of union. This formula shows that *artiads*, or even-ads, can produce only *artiads*. But any perissid, or odd-ad (except a monad), can by repetition produce a relative of any *adinity*.

Since the principal object of a notation for relatives is not to produce a handy *calculus* for the solution of special logical problems, but to help the study of logical principles, the study of logical graphs from that point of view must be postponed to a future occasion. For present purposes that notation is best which carries analysis the furthest, and presents the smallest number of unanalyzed forms. It will be best, then, to use single letters for relatives of some one definite and odd number of blanks. We

naturally choose three as the smallest number which will answer the purpose.

We shall, therefore, substitute for such a dyad as "—is lover of—" some such triad as "—is coexistent with , and a lover of—." If, then, we make –*w*– to signify "—is coexistent with , and with —," that which we have hitherto written as in Fig. 12 will be written as in Fig. 21. But having once recognised that such a mode

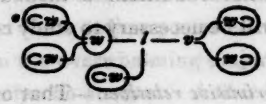

Fig. 21.

of writing is possible, we can continue to use our former methods, provided we now consider them as abbreviations.

The logical doctrine of this section, must, we may remark, find its application in metaphysics, if we are to accept the Kantian principle that metaphysical conceptions mirror those of formal logic.

§ 6. *Relatives of Second Intention.*—The general method of graphical representation of propositions has now been given in all its essential elements, except, of course, that we have not, as yet, studied any truths concerning special relatives; for to do so would seem, at first, to be "extralogical." Logic in this stage of its development may be called *paradisaical logic*, because it represents the state of Man's cognition before the Fall. For although, with this apparatus, it is easy to write propositions necessarily true, it is absolutely impossible to write any which is necessarily false, or, in any way which that stage of logic affords, to find out that anything is false. The mind has not as yet eaten of the fruit of the Tree of Knowledge of Truth and Falsity. Probably it will not be doubted that every child in its mental development necessarily passes through a stage in which he has some ideas, but yet has never recognised that an idea may be erroneous; and a stage that every child necessarily passes through must have been formerly passed through by the race in its adult development. It may be doubted whether many of the lower animals have any clear and

steady conception of falsehood; for their instincts work so unerringly that there is little to force it upon their attention. Yet plainly without a knowledge of falsehood no development of discursive reason can take place.

This paradisaical logic appears in the study of non-relative formal logic. But *there* no possible avenue appears by which the knowledge of falsehood could be brought into this Garden of Eden except by the arbitrary and inexplicable introduction of the Serpent in the guise of a proposition necessarily false. The logic of relatives, affords such an avenue, and *that*, the very avenue by which in actual development, this stage of logic supervenes. It is the avenue of experience and logical reflexion.

By *logical* reflexion, I mean the observation of thoughts in their expressions. Aquinas remarked that this sort of reflexion is requisite to furnish us with those ideas which, from lack of contrast, ordinary external experience fails to bring into prominence. He called such ideas *second intentions*. It is by means of *relatives of second intention* that the general method of logical representation is to find completion.

Let $\prec$ signify that "—is $\begin{cases} \text{neither} \\ \text{nor} \end{cases}$—." Then Fig. 22 means

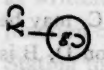

Fig. 22.

Fig. 23.

Fig. 24.

that taking any two things whatever, either the one is neither itself nor the other (putting it out of the question as an absurdity), or the other is a non-giver of something to that thing. That is, nothing gives all things, each to itself. Thus, the existence of any gen-

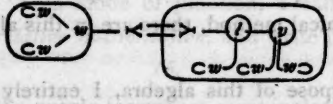

Fig. 25.

eral description of thing can be denied. Either medad of Fig. 23 means no wise men are virtuous. Fig. 24 is equivalent to Fig. 7. Fig. 25 means "each wise man is a lover of something virtuous."

Thus we see that this mode of junction,—lover of some virtuous,—which seems so simple,—is really complex. Fig. 26 means "some

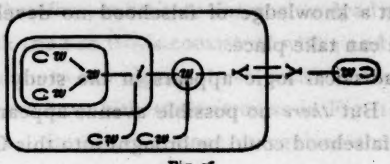

Fig. 26.

one thing is loved by all wise men." Fig. 27 means that every man is either wise or virtuous. Fig. 28 means that every man is both wise and virtuous.

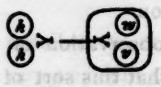

Fig. 27.

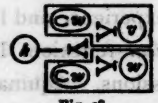

Fig. 28.

These explanations need not be carried further to show that we have here a perfectly efficient and highly analytical method of representing relations.

§ 7. *The Algebra of Dyadic Relatives.*—Although the primitive relatives are triadic, yet they may be represented with but little violence by means of dyadic relatives, provided we allow several attachments to one blank. For instance, A gives B to C, may be represented by saying A is the first party in the transaction D, B is subject of D, C is second party of D, D is a giving by the first party of the subject to the second party. Triadic relatives cannot conveniently be represented on one line of writing. These considerations led me to invent the algebra of dyadic relatives as a tolerably convenient substitute in many cases for the graphical method of representation. In place of the one "operation," or mode of conjunction of graphical method, there are in this algebra four operations.

For the purpose of this algebra, I entirely discard the idea that every compound relative consists of an antecedent and a consequent part. I consider the circle round the antecedent as a mere sign of negation, for which in the algebra I substitute an *obelus* over that antecedent. The line between antecedent and consequent, I

treat as a sign of an "operation" by itself. It signifies that anything whatever being taken as correlate of the first written member,—antecedent or consequent,—and as first relate of the second written member, either the one or the other is to be accepted. Thus in place of the relative of Fig. 29 signifying that "taking anything whatever, M, either—is not a lover of M, or M is a benefactor of —," that is "— is a lover only of a benefactor of —," I write

$$l \dagger b.$$

Or if it happens to be read the other way, putting a short mark over any letters to signify that relate and correlate are interchanged, I write the same thing

$$\breve{b} \dagger \breve{l}.$$

This operation, which may, at need, be denoted by a dagger in print, to which I give a scorpion-tail curve in its cursive form, I call *relative addition*.

The relative "— stands to everything which is a benefactor of — in the relation of servant of every lover of his," shows,

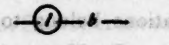

Fig. 29.

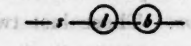

Fig. 30.

as written in Fig. 30, an unencircled bond between s and l. The junction of the l and the b may therefore be regarded as direct. Stating the relative so as to make this direct junction prominent, it is "— is servant of everything that is a lover of a benefactor of —." In the algebra, as far as already explained, "lover of a benefactor" would be written

$$\overline{l \dagger b}$$

that is, not a non-lover of every benefactor, or not a lover only of non-benefactors. This mode of junction, I call, in the algebra, the operation of *relative multiplication*, and write it

$$lb.$$

We have, then, the purely formal, or meaningless, equation

$$lb = \overline{l \dagger b}.$$

And in like manner, as a consequence of this,

$$l \dagger b = \overline{lb}.$$

That is to say, "To say that A is a lover of everything but benefactors of B," or "A is a non-lover only of benefactors of B," is the same as to say that A is not a non-lover of a non-benefactor of B.

To express in the algebra the relative of Fig. 31

Fig. 31.

or "—is both a lover and a benefactor of—," I write

$$l \cdot b,$$

calling this "the operation of *non-relative multiplication*." To express "—is either a lover or a benefactor of—," which might be written

$$\overline{\bar{l} \cdot \bar{b}},$$

I write

$$l \dotplus b,$$

calling this the operation of *non-relative addition*, or more accurately, of *aggregation*. These last two operations belong to the Boolian algebra of non-relative logic. They are De Morgan's operations of composition and aggregation. Boole himself did not use the last, but in place of it an operation more properly termed addition which gives no interpretable result when the aggregants have any common aggregant. Mr. Venn still holds out for Boole's operation, and there are weighty considerations in its favor. In my opinion, the decision between the two operations should depend upon whether the quantified predicate is rejected (when aggregation should be used), or accepted (when Boole's strict addition should be used).

The use of these four operations necessitates continual resort to parentheses, brackets, and braces to show how far the different compound relatives extend. It also becomes desirable to have a "copula of inclusion," or the sign of "is exclusively (if anything)." For this purpose I have since 1870 employed the sign $-\!<$ (intended for an improved $\leq$). It is easily made in the composing room from a dash followed by $<$, and in its cursive form is struck off in

two rapid strokes, thus $\prec$. Its meaning is exemplified in the formula

$$w \prec v$$

"anybody who is wise (if any there be) is exclusively found among the virtuous." We also require in this algebra the signs of relatives of second intention

0, "— is inconsistent with —," ∞, "— is coexistent with —,"
T, "— is other than —," I, "— is identical with."

The algebra has a moderate amount of power in skilful hands; but its great defect is the vast multitude of purely formal propositions which it brings along. The most significant of these are

$$s(l \fatsemi b) \prec sl \fatsemi b$$

and

$$(l \fatsemi b)s \prec l \fatsemi bs.$$

That is, whatever is a servant of something which is a lover of everything but benefactors is a servant-of-a-lover to everything but benefactors, etc.

Professor Schröder attaches, as it seems to me, too high a value to this algebra. That which is in his eyes the greatest recommendation of it is to me scarcely a merit, namely that it enables us to express in the outward guise of an equation propositions whose real meaning is much simpler than that of an equation.

§ 8. *General algebra of logic.*—Besides the algebra just described, I have invented another which seems to me much more valuable. It expresses with the utmost facility everything which can be expressed by a graph, and frequently much more clearly than the unabridged graphs described above. The method of using it in the solution of special problems has also been fully developed by me.

In this algebra every proposition consists of two parts, its quantifiers and its Boolian. The Boolian consists of a number of relatives united by a non-relative multiplication and aggregation. No relative operations are required (though they can be introduced if desired). Each elementary relative is represented by a letter on the line of writing with subjacent indices to denote the hecceities

which fill its blanks. An obelus is drawn over such a relative to deny it.

To the left of the Boolian are written the quantifiers. Each of these is a Π or a Σ with one of the indices written subjacent to it, to signify that in the Boolian every object in the universe is to be imaged substituted successively for that index and the non-relative product (if the quantifier is Π) or the aggregate (if the quantifier is Σ) of the results taken. The order of the quantifiers is, of course, material. Thus

$$\Pi_i \Sigma_j l_{ij} = (l_{11} \Psi l_{12} \Psi l_{13} \Psi \text{ etc.}) \cdot (l_{21} \Psi l_{22} \Psi l_{23} \Psi \text{ etc.}) \cdot \text{ etc.}$$

will mean anything loves something. But

$$\Sigma_j \Pi_i l_{ij} = l_{11} \cdot l_{21} \cdot l_{31} \cdot \text{ etc.} \Psi l_{12} \cdot l_{22} \cdot l_{32} \cdot \text{ etc.} \Psi l_{13} \cdot l_{23} \cdot l_{33} \cdot \text{ etc.} \Psi \text{ etc.}$$

will mean something is loved by all things.

This algebra, which has but two operations, and those easily manageable, is, in my opinion, the most convenient apparatus for the study of difficult logical problems, although the graphical method is capable of such modification as to render it substantially as convenient on the average. Nor would I refuse to avail myself of the algebra of dyadic relatives in the simpler cases in which it is easily handled.

§ 9. *Method of Calculating with the General Algebra.*—My rules for working this algebra, the fruit of long experience with applying it to a great variety of genuine inquiries, have never been published. Nor can I here do more than state such as the beginner will be likely to require.

A number of premises being given, it is required to know the most important conclusions of a certain description which can be drawn from them. The first step will be to express the premises by means of the general algebra, taking care to use entirely different letters *as indices* in the different premises.

These premises are then to be copulated (or, in Whewell's phrase, colligated), i. e., non-relatively multiplied together, by multiplying their Boolians and writing before the product all the quantifiers. The relative order of the quantifiers of each premise

must (in general) be undisturbed; but the relative order of quantifiers of different premises is arbitrary. The student ought to place Σ's as far to the left and Π's as far to the right as possible. Different arrangements of the quantifiers will lead to different conclusions from the premises. It sometimes happens that each of several arrangements leads to a conclusion which could not easily be reached from any other arrangement.

The premises, being so copulated, become one copulated premise. This copulated premise is next to be logically multiplied into itself any number of times, the indices being different in all the different factors. For there will be certain conclusions which I call conclusions of the first order, which can be drawn from the copulated premise without such involution, certain others, which I call inferences of the second order, which can be drawn from its square, etc. But after involution has been carried to a certain point, higher powers will only lead to inferences of subsidiary importance. The student will get a just idea of this matter by considering the rise and decline of interest in the theorems of any mathematical theory, such as geometry or the theory of numbers, as the fundamental hypotheses are applied more and more times in the demonstrations. The number of factors in the copulated premise, which embraces *all* the hypotheses that either theory assumes, is not great. Yet from this premise many thousand conclusions have already been drawn in the case of geometry and hundreds in the case of the theory of numbers. New conclusions are now coming in faster than ever before. From the nature of logic they can never be exhausted. But as time goes on the conclusions become more special and less important. It is true that mathematics, as a whole, does not become more special nor its late discoveries less important, because there is a growth of the hypotheses. Up to a certain degree, the importance of the conclusions increases with their "order." Thus, in geometry, there is nothing worth mention of the first order, and hardly of the second. But there is a great falling off in the importance of conclusions in the theories mentioned long before the fiftieth order has been reached.

This involution having been performed, the next step will be

the identification (occasionally the diversification) of certain indices. The rule is, that any index quantified with a Π can be transmitted, throughout the Boolian, into any other index whose quantifier stands to the left of its own, which now becomes useless, since it refers to nothing in the Boolian. For example, in

$$\Sigma_i \Pi_j l_{ij}$$

which in the Algebra of Dyadic Relatives would be written $\infty(l \mathfrak{z} 0)$, we can identify $\mathfrak{z}$ with i and write

$$\Sigma_i l_{ii}$$

which in the other algebra becomes $\infty(l \cdot 1)\infty$.

That done, the Boolian is to be manipulated according to any of the methods of non-relative Boolian algebra, and the conclusion is read off.

But it is only in the simplest cases that the above operations suffice. Relatives of second intention will often have to be introduced; and their peculiar properties must be attended to. Those of 0 and ∞ are covered by the rules of non-relative Boolian algebra; but it is not so with 1 and T. We have, for example, to observe that

$$\Pi_i x_i \dagger y_i = \Pi_i \Pi_j x_i \dagger \mathsf{T}_{ij} \dagger y_i.$$

$$\Sigma_i x_i \cdot y_i = \Sigma_i \Sigma_j x_i \cdot 1_{ij} \cdot y_j.$$

Exceedingly important are the relatives signifying "— is a quality of —" and "— is a relation of — to —." It may be said that mathematical reasoning (which is the only deductive reasoning, if not absolutely, at least eminently) almost entirely turns on the consideration of abstractions as if they were objects. The protest of nominalism against such hypostatisation, although, if it knew how to formulate itself, it would be justified as against much of the empty disputation of the medieval Dunces, yet, as it was and is formulated, is simply a protest against the only kind of thinking that has ever advanced human culture. Nobody will work long with the logic of relatives,—unless he restricts the problems of his studies very much,—without seeing that this is true.

§ 10. *Schröder's Conception of Logical Problems.*—Of my own labors in the logic of relatives since my last publication in 1884, I intend to give a slight hint in § 13. But I desire to give some idea

of a part of the contents of Schröder's last volume. In doing so, I shall adhere to my own notation; for I cannot accept Professor Schröder's proposed innovations. I shall give my reasons in detail for this dissent in the *Bulletin of the American Mathematical Society*. I will here only indicate their general nature. I have no objection whatever to the creation of a new system of signs *ab ovo*, if anybody can propose such a system sufficiently recommending itself. But *that* Professor Schröder does not attempt. He wishes his notation to have the support of existing habits and conventions, while proposing a measure of reform in the present usage. For that he must obtain general consent. Now it seems to me quite certain that no such general agreement can be obtained without the strictest deference to the principle of priority. Without that, new notations can only lead to confusion thrice confounded. The experience of biologists in regard to the nomenclature of their genera and other groups shows that this is so. I believe that their experience shows that the only way to secure uniformity in regard to conventions of this sort, is to accept for each operation and relative the sign definitively recommended by the person who introduced that operation or relative into the Boolian algebra, unless there are the most *substantial* reasons for dissatisfaction with the meaning of the sign. Objections of lesser magnitude may justify slight modifications of signs; as I modify Jevons's $+$ to $\dot{+}$, by uniting the two dots by a connecting line, and as I so far yield to Schröder's objections to using ∞ for the sign of whatever is, as to resort to the similarly shaped sign of Aries Υ (especially as a notation of some power is obtained by using all the signs of the Zodiac in the same sense, as I shall show elsewhere). In my opinion, Professor Schröder alleges no sufficient reason for a single one of his innovations; and I further consider them as *positively* objectionable.

The volume consists of thirty-one long sections filling six hundred and fifty pages. I can, therefore, not attempt to do more than to exemplify its contents by specimens of the work selected as particularly interesting. Professor Schröder chiefly occupies himself with what he calls "solution-problems," in which it is required to deduce from a given proposition an *equation* of which one mem-

ber consists in a certain relative determined in advance, while the other member shall not contain that relative. He rightly remarks that such problems often involve problems of elimination.

While I am not at all disposed to deny that the so-called "solution-problems," consisting in the ascertainment of the general forms of relatives which satisfy given conditions, are often of considerable importance, I cannot admit that the interest of logical study centres in them. I hold that it is usually much more to the purpose to express in the simplest way what a given premise discloses in regard to the constitution of a relative, whether that simplest expression is of the nature of an equation or not. Thus, one of Schröder's problems is, "Given $x \prec\!\!\prec a$, required x,"—for instance, knowing that an opossum is a marsupial, give a description of the opossum. The so-called solution is $\sum_u x u \cdot a$, or opossums embrace precisely what is common to marsupials and to some other class. In my judgment $x \prec\!\!\prec a$ might with great propriety be called the solution of $\sum_u x u \cdot a$. When the information contained in a proposition is not of the nature of an equation, why should we, by circumlocutions, insist upon expressing it in the form of an equation?

Professor Schröder attaches great importance to the generality of solutions. In my opinion, this is a mistake. It is not merely that he insists that solutions shall be *complete*, as for example when we require *every root* of a numerical equation, but further that they shall all be embraced under one algebraical expression. Upon that he insists and with that he is satisfied. Whether or not the "solution" is such as to exhibit anything of the real constitution of the relative which forms the first member of the equation he does not seem to care; at least, there is no apparent consideration of the question of how such a result can be secured.

Pure mathematics always selects for the subjects of its studies manifolds of perfect homogeneity; and thence it comes that for the problems which first present themselves general solutions are possible, which notwithstanding their generality, guide us at once to all the particular solutions. But even in pure mathematics the class of problems which are capable of solutions at once general

and useful is an exceedingly limited one. All others have to be treated by subdivision of cases. That is what meets us everywhere in higher algebra. As for general solutions, they are for the most part trivial,—like the well-known and obvious test for a prime number that the continued product of all lesser numbers increased by 1 shall be divisible by that number. Only in those cases in which a general solution points the way to the particular solutions is it valuable; for it is only the particular solutions which picture to the mind the solution of a problem; and a form of words which fails to produce a definite picture in the mind is meaningless.

Professor Schröder endeavors to give the most general formula of a logical problem. It is in dealing with such very general and fundamental matters that the exact logician is most in danger of violating his own principles of exactitude. To seek a formula for all logical problems is to ask what it is, in general terms, that men inquire. To answer that question, my own logical proceeding would be to note that it asks what the essence of a question, in general, is. Now a question is a rational contrivance or device, and in order to understand any rational contrivance, experience shows that the best way is to begin by considering what circumstances of need prompted the contrivance, and then upon what general principle its action is designed to fill that need. Applying this general experience to the case before us, we remark that every question is prompted by some need,—that is, by some unsatisfactory condition of things, and that the object of asking the question is to fill that need by bringing reason to bear upon it and to do this by a hypnotically suggestive indication of that to which the mind has to apply itself. I do not know that I have ever, before this minute, considered the question what is the most general formulation of a problem in general; for I do not find much virtue in general formulæ. Nor do I think my answer to this question affords any particularly precious suggestion. But its ordinary character makes it all the better an illustration of the manner—or one of the manners—in which an exact logician may attack, off-hand, a suddenly sprung question. A question, I say, is an indication suggestive (in the hypnotic sense) of what has to be thought about in order to satisfy

some more or less pressing want. Ideas like those of this statement, and not talk about φx, and "roots," and the like, must, in my opinion, form the staple of a logical analysis and useful description of a problem, in general. I am none the less a mathematical logician for that. If of two students of the theory of numbers one should insist upon considering numbers as expressed in a system of notation like the Arabic (though using now one number as base of the numeration, and now another), while the other student should maintain that all that was foreign to the theory of numbers, which ought not to consider upon what system the numbers with which it deals are expressed, those two students would, to my apprehension, occupy positions analogous to that of Schröder and mine in regard to this matter of the formulation of the problems of logic; and supposing the student who wished to consider the forms of expression of numbers were to accuse the other of being wanting in the spirit of an arithmetician, that charge would be unjust in quite the same way in which it would be unjust to charge me with deficiency in the mathematical spirit on account of my regarding the conceptions of "values," and "roots," and all that as very special ideas, which can only lumber up the field of consciousness with such hindrances as it is the very end and aim of that diagrammatic method of thinking that characterises the mathematician to get rid of.

But different questions are so very unlike that the only way to get much idea of the nature of a problem is to consider the different cases separately. There are in the first place questions about needs and their fulfillment which are not directly affected by the asking of the questions. A very good example is a chess problem. You have only to experiment in the imagination just as you would do on the board if it were permitted to touch the men, and if your experiments are intelligently conducted and are carried far enough, the solution required must be discovered. In other cases, the need to which the question relates is nothing but the intellectual need of having that question answered. It may happen that questions of this kind can likewise be answered by imaginary experimentation; but the more usual case requires real experimentation. The need

is of one or other of two kinds. In the one class of cases we experience on several occasions to which our own deliberate action gave a common character, an excitation of one and the same novel idea or sensation, and the need is that a large number of propositions having the same novel consequent but different antecedents, should be replaced by one proposition which brings in the novel element, so that the others shall appear as mere consequences of every day facts with a single novel one. We may express this intellectual need in a brief phrase as the need of synthetising a multitude of subjects. It is the need of *generalisation*. In another class of cases, we find in some new thing, or new situation, a great number of characters, the same as would naturally present themselves as consequences of a hypothetical state of things, and the need is that the large number of novel propositions with one subject or antecedent should be replaced by a single novel proposition, namely that the new thing or new occasion belongs to the hypothetical class, from which all those other novelties shall follow as mere consequences of matters of course. This intellectual need, briefly stated, is the need of synthetising a multitude of predicates. It is the need of *theory*. Every problem, then, is either a problem of consequences, a problem of generalisation, or a problem of theory. This statement illustrates how special solutions are the only ones which directly mean anything or embody any knowledge; and general solutions are only useful when they happen to suggest what the special solutions will be.

Professor Schröder entertains very different ideas upon these matters. The general problem, according to him, is, "Given the proposition $Fx = 0$, required the 'value' of x_0," that is, an expression not containing x which can be equated to x. This 'value' must be the "general root," that is, it must, under one general description, cover every possible object which fulfils a given condition. This, by the way, is the simplest explanation of what Schröder means by a "solution-problem"; it is the problem to find that form of relative which necessarily fulfils a given condition and in which every relative that fulfils that condition can be expressed. Schröder shows that the solution of such a problem can be put into

the form $\sum_u [x = fu]$, which means that a suitable logical function (f) of *any* relative, u, no matter what, will satisfy the condition $Fx = 0$; and that nothing which is not equivalent to such a function will satisfy that condition. He further shows, what is very significant, that the solution may be required to satisfy the "adventitious condition" $fx = x$. This fact about the adventitious condition is all that prevents me from rating the value of the whole discussion as far from high.

Professor Schröder next produces what he calls "the rigorous solution" of the general question. This promises something very fine,—the rigorously correct resolution of everything that ever could (but for this knowledge) puzzle the human mind. It is true that it supposes that a particular relative has been found which shall satisfy the condition $Fx = 0$. But that is seldom difficult to find. Either 0, or ∞, or some other trivial solution commonly offers itself. Supposing, then, that a be this particular solution, that is, that $Fa = 0$, the "rigorous solution" is

$$x = fu = a \cdot {}^{\infty}(Fu)^{\infty} \Psi u \cdot (0 \,\lambda\, \overline{Fu} \,\lambda\, 0).$$

That is, it is such a function of u that when u satisfies the condition $Fu = 0$, $fu = u$; but when u does not satisfy this condition $fu = a$. Now $Fa = 0$.

Since Professor Schröder carries his algebraicity so very far, and talks of "roots," "values," "solutions," etc., when, even in my opinion, with my bias towards algebra, such phrases are out of place, let us see how this "rigorous solution" would stand the climate of numerical algebra. What should we say of a man who professed to give rigorous general solutions of algebraic equations of every degree (a problem included, of course, under Professor Schröder's general problem)? Take the equation $x^5 + Ax^4 + Bx^3 + Cx^2 + Dx + E = 0$. Multiplying by $x - a$ we get

$$x^6 + (A - a)x^5 + (B - aA)x^4 + (C - aB)x^3 + (D - aC)x^2 + (E - aD)x - aE = 0$$

The roots of this equation are precisely the same as those of the proposed quintic together with the additional root $x = a$. Hence, if we solve the sectic we thereby solve the quintic. Now, our

Schröderian solver would say, "There is a certain function, fu, every value of which, no matter what be the value of the variable, is a root of the sextic. And this function is formed by a direct operation. Namely, for all values of u which satisfy the equation

$$u^6+(A-a)u^5+(B-aA)u^4+(C-aB)u^3+(D-aC)u^2+(E-aD)u-aE=0$$

$fu=u$, while for all other values, $fu=a$.

Then, $x=fu$ is the expression of every root of the sextic and of nothing else. It is safe to say that Professor Schröder would pronounce a pretender to algebraical power who should talk in that fashion to be a proper subject for *surveillance* if not for confinement in an asylum. Yet he would only be applying Professor Schröder's "rigorous solution," neither more nor less. It is true that Schröder considers this solution as somewhat unsatisfactory; but he fails to state any principle according to which it should be so. Nor does he hold it too unsatisfactory to be frequently resorted to in the course of the volume. The *invention* of this solution exhibits in a high degree that very effective ingenuity which the *solution itself* so utterly lacks, owing to its resting on no correct conception of the nature of problems in general and of their solutions and of the meaning of a proposition.

§ 11. *Professor Schröder's Pentagrammatical Notation.*—Professor Schröder's greatest success in the logic of relatives, is due precisely to his having, in regard to certain questions, proceeded by the separation of cases, quite abandoning the glittering generalities of the algebra of dyadic relatives. As his greatest success, I reckon his solutions of "inverse row and column problems" in § 16, resting upon an investigation in § 15 of the relations of various compound relatives which end in 0, ∞, 1, and T. The investigations of § 15 might perfectly well have been carried through without any other instrument than the algebra of dyadic relatives. This course would have had certain advantages, such as that of exhibiting the principles on which the formulæ rest. But directness of proof would not have been of the number of those advantages; this is on the contrary decidedly with the notation invented and used by Professor Schröder. This notation may be called *pentagrammatic*, since it

denotes a relative by a row of 5 characters. Imagine a list to be made of all the objects in the universe. Second, imagine a switch-board, consisting of a horizontal strip of brass for each object (these strips being fastened on a wall at a little distance one over another according to the order of the objects in the list) together with a vertical strip of brass for each object (these strips being fastened a little forward of the others, and being arranged in the same order), with holes at all the intersections, so that when a brass plug is inserted in any hole, the object corresponding to the horizontal brass strip can act in some way upon the object corresponding to the vertical brass strip. In order then, by means of this switch-board, to get an analogue of any dyadic relative, a lover of —," we insert plugs so that A and B, being any two objects, A can act on B, if and only if A is a lover of B. Now in Professor Schröder's pentagrammatic notation, the first of the five characters denoting any logical function of a primitive relative, a, refers to those horizontal strips, all whose holes are plugged in the representation of a (or, as we may say for short, "in a"), the second refers to those horizontal strips, each of which has in a every hole plugged but one. This one, not necessarily the same for all such strips, may be denoted by A. The third character refers to those horizontal strips which in a have several holes plugged, and several empty. The full holes (different, it may be, in the different horizontal strips) may be denoted by β. The fourth character refers to those horizontal strips which in a have, each of them, but one hole plugged, generally a different hole in each. This one plugged hole may be denoted by Γ. The fifth character will refer to those rows each of which in a has all its holes empty. Then, a will be denoted by $\infty\,\bar{\mathrm{A}}\,\beta\,\Gamma\,0$; and $\bar{a}$ by $0\,\mathrm{A}\,\bar{\beta}\,\bar{\Gamma}\,\infty$; for in $\bar{a}$, all the holes must be filled that are void in a, and *vice versa*. Consequently $\bar{a}\,\mathsf{T} = 0\,\bar{\mathrm{A}}\,\infty\,\infty\,\infty$. This shall be shown as soon as we have first examined the pentagrammatic symbol for a. This symbol divides a into four aggregants, viz:

$$a = (a \between 0) \dagger a \cdot [(a \between 1) \cdot \bar{a}]\,\mathsf{T} \dagger a \cdot a\,\mathsf{T} \cdot (\bar{a} \cdot \bar{a}\,\mathsf{T})\,\mathsf{T} \dagger a \cdot (\bar{a} \between 1)$$

In order to prove, by the algebra itself that this equation holds, we remark that $a = a \cdot b \dagger a \cdot \bar{b}$, whatever b may be. For b, substitute

$(a \mathbin{\text{⨟}} 0)$. Then, $a \mathbin{\text{⨟}} 0 \prec a \mathbin{\text{⨟}} \mathsf{T}$; but $a \mathbin{\text{⨟}} \mathsf{T} = a$. Hence, $a \cdot b = a \mathbin{\text{⨟}} 0$. $a \cdot \bar{b} = a \cdot \bar{a} \mathbin{\text{℘}} = a \cdot \bar{a}(\mathsf{I} \dagger \mathsf{T}) = a \cdot (\bar{a}\mathsf{I} \dagger \bar{a}\mathsf{T})$. But $\bar{a}\mathsf{I} = \bar{a}$, and $a \cdot \bar{a} = 0$. Hence $a \cdot \bar{b} = a \cdot \bar{a}\mathsf{T}$. Thus $a = a \mathbin{\text{⨟}} 0 \dagger a \cdot \bar{a}\mathsf{T}$. Now, in $\bar{a} = \bar{a} \cdot c \dagger \bar{a} \cdot \bar{c}$, substitute for c, $a \mathbin{\text{⨟}} \mathsf{I}$. This gives $\bar{a} = (a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a} \dagger \bar{a}\mathsf{T} \cdot \bar{a}$; and thus, $a = a \mathbin{\text{⨟}} 0 \dagger a \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \dagger a \cdot (\bar{a}\mathsf{T} \cdot \bar{a})\mathsf{T}$. Finally, $a = a \cdot a\mathsf{T} \dagger a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I})$. But $a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) = a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (\bar{a}\mathsf{T} \cdot \bar{a})\mathsf{T} \dagger a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot \{[(a \mathbin{\text{⨟}} \mathsf{I}) \dagger a] \mathbin{\text{⨟}} \mathsf{I}\}$.

And

$$
\begin{aligned}
a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot \{[(a \mathbin{\text{⨟}} \mathsf{I}) \dagger a] \mathbin{\text{⨟}} \mathsf{I}\} &= a \cdot \{\bar{a} \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \dagger a] \mathbin{\text{⨟}} \mathsf{I}\} && \text{(by distribution)} \\
&= a \cdot [\bar{a} \cdot (a \mathbin{\text{⨟}} \mathsf{I}) \mathbin{\text{⨟}} \mathsf{I}] && \text{(since } \bar{a} \cdot a = 0\text{)} \\
&= a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} \mathsf{I} \mathbin{\text{⨟}} \mathsf{I}) && \text{(by distribution)} \\
&= a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} 0) && \text{(if more than 2 things exist)} \\
&= a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} \mathsf{I} \cdot \mathsf{T}) && \text{(since } 0 = \mathsf{I} \cdot \mathsf{T}\text{)} \\
&= a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} \mathsf{T}) && \text{(by distribution)} \\
&= a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (a \mathbin{\text{⨟}} \mathsf{I}) && \text{(since } a \mathbin{\text{⨟}} \mathsf{T} = a\text{)} \\
&= a \cdot (\bar{a} \cdot a \mathbin{\text{⨟}} \mathsf{I}) && \text{(by distribution)} \\
&= a \cdot (0 \mathbin{\text{⨟}} \mathsf{I}) && \text{(since } \bar{a} \cdot a = 0\text{)} \\
&= a \cdot 0 && \text{(if more than 1 object exists)} \\
&= 0.
\end{aligned}
$$

So that $a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) = a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}) \cdot (\bar{a}\mathsf{T} \cdot \bar{a})\mathsf{T}$ and thus

$$a = a \mathbin{\text{⨟}} 0 \dagger a \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \dagger a \cdot a\mathsf{T}(\bar{a}\mathsf{T} \cdot \bar{a})\mathsf{T} \dagger a \cdot (\bar{a} \mathbin{\text{⨟}} \mathsf{I}).$$

This is the meaning of the symbol $\text{℘}\,\overline{\mathrm{A}}\beta\Gamma 0$.

We, now, at length, return, as promised to the examination of $\bar{a}\mathsf{T}$. First, $a \mathbin{\text{⨟}} 0 \prec \overline{\bar{a}\mathsf{T}} \mathbin{\text{⨟}} 0$. For $\overline{\bar{a}\mathsf{T}} = a \mathbin{\text{⨟}} \mathsf{I}$ and $a \mathbin{\text{⨟}} \mathsf{I} \mathbin{\text{⨟}} 0 = a \mathbin{\text{⨟}} (\mathsf{I} \mathbin{\text{⨟}} 0) = a \mathbin{\text{⨟}} 0$. Hence the first character in the pentagrammatic symbol for $\bar{a}\mathsf{T}$ must be 0. Second $a \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \prec \bar{a}\mathsf{T} \cdot [(\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}) \cdot \overline{\bar{a}\mathsf{T}}]\mathsf{T}$. For it is plain that $a \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \prec [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \prec \bar{a}\mathsf{T}$. Also $\bar{a} \prec \bar{a} \mathbin{\text{℘}} \prec \bar{a}(\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}) \prec \bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}$. Hence $[(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \prec [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot (\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I})]\mathsf{T}$. But $a \mathbin{\text{⨟}} \mathsf{I} = \overline{\bar{a}\mathsf{T}}$. Hence, $a \cdot [(a \mathbin{\text{⨟}} \mathsf{I}) \cdot \bar{a}]\mathsf{T} \prec \bar{a}\mathsf{T} \cdot [(\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}) \cdot \overline{\bar{a}\mathsf{T}}]\mathsf{T}$. Hence, the second character in the pentagrammatic sign for $\bar{a}\mathsf{T}$, is the same as that of a. Thirdly $a \cdot a\mathsf{T} \cdot (\bar{a}\mathsf{T} \cdot \bar{a})\mathsf{T} \prec \bar{a}\mathsf{T} \mathbin{\text{⨟}} 0$. For $\bar{a} \prec \bar{a}\mathsf{I} \prec \bar{a}(\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}) \prec \bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}$. Hence $(\bar{a} \cdot \bar{a}\mathsf{T})\mathsf{T} \prec [(\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I}) \cdot (\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{T})]\mathsf{T} \prec (\bar{a}\mathsf{T} \mathbin{\text{⨟}} \mathsf{I} \cdot \mathsf{T})\mathsf{T} \prec (\bar{a}\mathsf{T} \mathbin{\text{⨟}} 0)\mathsf{T} \prec \bar{a}\mathsf{T} \mathbin{\text{⨟}} 0\mathsf{T} \prec \bar{a}\mathsf{T} \mathbin{\text{⨟}} 0$. Consequently, the third character of the pentagrammatic symbol of $\bar{a}\mathsf{T}$ must be ℘.

Fourthly, $a:(\bar{a}\,;1)\prec\bar{a}\,\mathsf{T}\,;0$. For we have just seen that $\bar{a}\prec\bar{a}\,\mathsf{T}\,;1$. Hence $\bar{a}\,;1\prec\bar{a}\,\mathsf{T}\,;1\,;1$. But $1\,;1=0$ if there is more than one object in the universe. Hence $\bar{a}\,;1\prec\bar{a}\,\mathsf{T}\,;0$. Consequently, the fourth character of the pentagrammatic formula for $\bar{a}\,\mathsf{T}$ is ∞. Finally, $\bar{a}\,;0\prec\bar{a}\,\mathsf{T}\,;0$. For $\bar{a}\,;0\prec\bar{a}\,;0\,;0\prec\bar{a}\,;1\cdot\mathsf{T}\,;0\prec(\bar{a}\,;1)\cdot(\bar{a}\,;\mathsf{T})\,;0\prec\bar{a}\,;1\,;0\prec\bar{a}\,\mathsf{T}\,;0$. Hence the fifth character of the pentagram of $\bar{a}\,\mathsf{T}$ is ∞. In fine, that pentagram is $0\bar{\mathrm{A}}\infty\infty\infty$. Professor Schröder obtains this result more directly by means of a special calculus of the pentagrammatic notation. In that way, he obtains, in § 15, a vast number of formulæ, which in § 16 are applied in the first place with great success to the solution of such problems as this: Required a form of relation in which everything stands to something but nothing to everything. The author finds instantaneously that every relative signifying such a relation must be reducible to the form $\bar{u}^{\infty}\cdot u\dotplus 1\cdot(u\,;0\dotplus\bar{u}\,;0)$. In fact, the first term of this expression $\bar{u}^{\infty}\cdot u$, for which $\bar{u}^{\infty}\cdot u^{\infty}$ might as well be written, embraces all the relatives in question. For let $\bar{u}$ be any such relative. Then, $u=\bar{u}^{\infty}\cdot u$. The second term is added, curiously enough, merely to *exclude other relations*. For if u is such a relative that something is u to everything or to nothing, then that something would be in the relation $\bar{u}^{\infty}\cdot u$ to nothing. To give it a correlate the second term is added; and since all the relatives are already included, it matters not what that correlate be, so long as the second term does not exclude any of the required relatives which are included under the first term. Let v be any relative of the kind required, then $v\cdot(u\,;0\dotplus\bar{u}\,;0)$ will answer for the second term. If we had no letter expressing a relation known to be of the required kind, the problem would be impossible. Fortunately, both 1 and T are of that kind. Of course, the negative of such a relative is itself such a relative; so that

$$(u\,;0\,\bar{u}\,;0)^{\cdot}(v\dotplus u^{\infty}\cdot\bar{u}^{\infty})$$

would be an equivalent form, equally with

$$(u\,;0\dotplus\bar{u}\,;0)\cdot v\dotplus u^{\infty}\cdot\bar{u}^{\infty}.$$

§ 16 concludes with some examples of eliminations of great apparent complexity. In the first of these we have given $x=$

$(\bar{u}\,\text{Ⅎ}\,1)^{\infty} \dagger u$; and it is required to eliminate u. We have, however, instantly $u \prec x$

$$(\bar{u}\,\text{Ⅎ}\,1)^{\infty} \prec x$$

Whence, immediately,

$$(\bar{x}\,\text{Ⅎ}\,1)^{\infty} \prec x,$$

or

$$^{\infty} \prec (x \cdot x\,\text{T})^{\infty}.$$

The next example, the most complicated, requires u to be eliminated from the equation

$$x = \bar{u}\,\text{Ⅎ}\,0 \dagger (u\,\text{Ⅎ}\,1)^{\infty} \cdot \bar{u}\,\text{T} \dagger (u\,\text{Ⅎ}\,1) \cdot \bar{u} \dagger (\bar{u}\,\text{Ⅎ}\,1) \cdot u \dagger (u\,\text{T} \cdot \bar{u}\,\text{T}\,\text{Ⅎ}\,0) \cdot \bar{u},$$

He performs the elimination by means of the pentagrammatic notation very easily as follows: Putting $u = {}^{\infty}\bar{\text{A}}\beta\Gamma 0$

$$\begin{aligned}
\bar{u}\,\text{Ⅎ}\,0 &= 0\,0\,0\,0\,{}^{\infty} \\
(u\,\text{Ⅎ}\,1)^{\infty} \cdot \bar{u}\,\text{T} &= 0\,\bar{\text{A}}\,0\,0\,0 \\
(u\,\text{Ⅎ}\,1) \cdot \bar{u} &= 0\,\text{A}\,0\,0\,0 \\
(\bar{u}\,\text{Ⅎ}\,1) \cdot u &= 0\,0\,0\,\Gamma\,0 \\
(u\,\text{T} \cdot \bar{u}\,\text{T}\,\text{Ⅎ}\,0) \cdot \bar{u} &= 0\,0\,\bar{\beta}\,0\,0 \\
\text{sum} \qquad & \overline{0\,{}^{\infty}\,\bar{\beta}\,\Gamma\,{}^{\infty}}
\end{aligned}$$

Thus, x is of the form ${}^{\infty}\,\bar{\beta}\,\Gamma\,0$, which has been found in former problems to imply $x\,\text{Ⅎ}\,1 \prec x$.

Without the pentagrammatical notation this elimination would prove troublesome, although with that as a guide it could easily be obtained by the algebra alone.

§ 12. *Professor Schröder's Iconic Solution of* $x \prec \varphi x$.

Another valuable result obtained by Professor Schröder is the solutions of the problem

$$x \prec \varphi x.$$

Namely, he shows that

$$x = f^{\infty} u$$

where

$$fu = u \cdot \varphi u$$

[Of course, by contraposition, this gives for the solution of $\varphi x \prec x$ $x = f^{\infty} u$ where $fu = u \dagger \varphi u$.] The correctness of this solution will appear upon a moment's reflexion; and nearly all the useful solutions in the volume are cases under this.

It happens very frequently that the iteration of the functional operation is unnecessary, because it has no effect.

Suppose, for example, that we desire the general form of a "transitive" relative, that is, such a one, x, that

$$x\,x \prec x.$$

In this case, since $1 \prec l \dagger l$ whatever l may be, we have

$$x \prec x\,1 \prec x\,(x \dagger \breve{x}) \prec x\,x \dagger \breve{x} \prec x \dagger \breve{x},$$

or

$$x \prec x \dagger \breve{x}$$

If, then,

$$fu = u \cdot (u \dagger \breve{u}),$$

we have

$$x = f^{\infty} u.$$

Here,

$$fu \prec u;$$

so that

$$f^{\infty} u \prec fu.$$

Also,

$$f^2 u = fu \cdot (fu \dagger \breve{fu}) = u \cdot (u \dagger \breve{u}) \cdot [u \cdot (u \dagger \breve{u}) \dagger (\breve{u} \Psi u \breve{u})]$$
$$= u \cdot (u \dagger \breve{u}) \cdot [u f(1 \Psi u)\breve{u}] \cdot [u \dagger \breve{u} \dagger (1 \Psi u)\breve{u}].$$

Now

$$fu = u \cdot (u \dagger \breve{u}) = u \cdot (u \dagger \breve{u}) \cdot (u \dagger \breve{u}) \cdot (u \dagger \breve{u}) = u \cdot (u \dagger \breve{u}) \cdot (u \dagger 1\breve{u}) \cdot (u \dagger 1\breve{u})$$
$$\prec u \cdot (u \dagger \breve{u}) \cdot [u \dagger (1 \Psi u)\breve{u}] \cdot [u \dagger (\breve{u} \dagger u)\breve{u}] \prec$$
$$\prec u \cdot (u \dagger \breve{u}) \cdot [u \dagger (1 \Psi u)\breve{u}] \cdot (u \dagger \breve{u} \dagger u\breve{u})$$
$$\prec u \cdot (u \dagger \breve{u}) \cdot [u \dagger (1 \Psi u)\breve{u}] \cdot [u \dagger \breve{u} \dagger (1 \Psi u)\breve{u}] \prec f^2 u.$$

Thus $fu = f^{\infty} u$; and

$$x = \sum_{u} u \cdot (u \dagger \breve{u})$$

This is a truly iconic result; that is, it shows us what the constitution of a transitive relative really is. It shows us that transitiveness always depends upon inclusion; for to say that A is $l \dagger l$ of B is to say that the things loved by B are included among those loved by A. The factor $u \dagger \breve{u}$ is transitive by itself; for

$$(u \dagger \breve{u})(u \dagger \breve{u}) \prec u \dagger \breve{u}\,u \dagger \breve{u} \prec u \dagger \mathsf{T} \dagger \breve{u} \prec u \dagger \breve{u}.$$

The effect of the other factor, u, of the form for the general transitive is merely in certain cases to exclude universal identity, and

thus to extend the class of relatives represented by $\alpha \breve{\beta} \beta$ so as to include those of which it is not true that $l \prec x$. Here we have an instance of restriction having the effect of extension, that is, restriction of special relatives extends the class of relatives represented. This does not take place in all cases, but only where certain relatives can be represented in more than one way.

Indicating, for a moment, the copula by a dash, the typical and fundamental syllogism is

$$\begin{array}{l} A—B \quad B—C \\ \therefore A—C. \end{array}$$

That is to say, the principle of this syllogism enters into every syllogism. But to say that this is a valid syllogism is merely to say that the copula expresses a transitive relation. Hence, when we now find that transitiveness always depends upon inclusion, the initial analysis by which the copula of inclusion was taken as the general one is fully confirmed. For the chief end of formal logic is the representation of the syllogism.

§ 13. *Introduction to the Logic of Quantity.*—The great importance of the idea of quantity in demonstrative reasoning seems to me not yet sufficiently explained. It appears, however, to be connected with the circumstance that the relations of being greater than and of being at least as great as are transitive relations. Still, a satisfactory evolutionary logic of mathematics remains a desideratum. I intend to take up that problem in a future paper. Meantime the development of projective geometry and of geometrical topics has shown that there are at least two large mathematical theories of continuity into which the idea of continuous *quantity*, in the usual sense of that word, does not enter at all. For projective geometry Schubert has developed an algebraical calculus which has a most remarkable affinity to the Boolian algebra of logic. It is, however, imperfect, in that it only gives imaginary points, rays, and planes, without deciding whether they are real or not. This defect cannot be remedied until topology—or, as I prefer to call it, mathematical topics—has been further developed and its logic accurately analysed. To do this ought to be one of the first tasks of exact logicians. But before that can be accomplished, a perfectly

satisfactory logical account of the conception of continuity is required. This involves the definition of a certain kind of infinity; and in order to make that quite clear, it is requisite to begin by developing the logical doctrine of infinite multitude. This doctrine still remains, after the works of Cantor, Dedekind, and others, in an inchoate condition. For example, such a question remains unanswered as the following: Is it, or is it not, logically possible for two collections to be so multitudinous that neither can be put into a one-to-one correspondence with a part or the whole of the other? To resolve this problem demands, not a mere *application* of logic, but a further *development* of the conception of logical possibility.

I formerly defined the possible as that which in a given state of information (real or feigned) we do not know not to be true. But this definition to-day seems to me only a twisted phrase which, by means of two negatives, conceals an anacoluthon. We know in advance of experience that certain things are not true, because we see they are impossible. Thus, if a chemist tests the contents of a hundred bottles for fluorine, and finds it present in the majority, and if another chemist tests them for oxygen and finds it in the majority, and if each of them reports his result to me, it will be useless for them to come to me together and say that they know infallibly that fluorine and oxygen cannot be present in the same bottle; for I see that such infallibility is *impossible*. I know it is not true, because I satisfy myself that there is no room for it even in that ideal world of which the real world is but a fragment. I need no sensible experimentation, because ideal experimentation establishes a much broader answer to the question than sensible experimentation could give. It has come about through the agencies of development that man is endowed with intelligence of such a nature that he can by ideal experiments ascertain that in a certain universe of logical possibility certain combinations occur while others do not occur. Of those which occur in the ideal world some do and some do not occur in the real world; but all that occur in the real world occur also in the ideal world. For the real world is the world of sensible experience, and it is a part of the process of sensible experience to locate its facts in the world of ideas. This

is what I mean by saying that the sensible world is but a fragment of the ideal world. In respect to the ideal world we are virtually omniscient; that is to say, there is nothing but lack of time, of perseverance, and of activity of mind to prevent our making the requisite experiments to ascertain positively whether a given combination occurs or not. Thus, every proposition about the ideal world can be ascertained to be either true or false. A description of thing which occurs in that world is *possible, in the substantive logical sense.* Very many writers assert that everything is logically possible which involves no contradiction. Let us call that sort of logical possibility, *essential*, or *formal*, logical possibility. It is not the only logical possibility; for in this sense, two propositions contradictory of one another may both be severally possible, although their combination is not possible. But in the *substantive* sense, the contradictory of a possible proposition is impossible, because we are virtually omniscient in regard to the ideal world. For example, there is no contradiction in supposing that only four, or any other number, of independent atoms exist. But it is made clear to us by ideal experimentation, that five atoms are to be found in the ideal world. Whether all five are to be found in the sensible world or not, to say that there are only four in the ideal world is a proposition absolutely to be rejected, notwithstanding its involving no contradiction.

It would be a great mistake to suppose that ideal experimentation can be performed without danger of error; but by the exercise of care and industry this danger may be reduced indefinitely. In sensible experimentation, no care can always avoid error. The results of induction from sensible experimentation are to afford some ratio of frequency with which a given consequence follows given conditions in the existing order of experience. In induction from ideal experimentation, no particular order of experience is forced upon us; and consequently no such numerical ratio is deducible. We are confined to a dichotomy: the result either is that some description of thing occurs or that it does not occur. For example, we cannot say that one number in every three is divisible by three and one in every five is divisible by five. This is, indeed,

so if we choose to arrange the numbers in the order of counting; but if we arrange them with reference to their prime factors, just as many are divisible by one prime as by another. I mean, for instance, when they are arranged as follows:

1,	2,	4,	8,	etc.	5,	10,	20,	40,	etc.	7,	14,	28,	56,	etc.	35,	70, etc.
3,	6,	12,	24,	etc.	15,	30,	60,	120,	etc.	21,	42,	84,	168,	etc.	105,	210, etc.
9,	18,	36,	72,	etc.	45,	90,	180,	360,	etc.			etc.			etc.	
27,	54,	108,	16,	etc.	135,	270,	540,	1080,	etc.							
		etc.					etc.									

Thus, dichotomy rules the ideal world. Plato, therefore, for whom that world alone was real, showed that insight into concepts but dimly apprehended that has always characterised philosophers of the first order, in holding dichotomy to be the only truthful mode of division. Lofty moral sense consists in regarding, not indeed *the*, but yet *an*, ideal world as in some sense the only real one; and hence it is that stern moralists are always inclined to dual distinctions.

Ideal experimentation has one or other of two forms of results. It either proves that $\Sigma_i\, m_i$, a particular proposition true of the ideal world, and going on, finds $\Sigma_i\, \overline{m}_i$ also true; that is, that m and $\overline{m}$ are both possible, or it succeeds in its induction and shows the universal proposition $\Pi_i\, m_i$ to be true of the ideal world; that is that $\overline{m}$ is *necessary* and m *impossible*.

Every result of an ideal induction clothes itself, in our modes of thinking, in the dress of a *contradiction*. It is an anacoluthon to say that a proposition is impossible *because* it is selfcontradictory. It rather is thought so as to appear selfcontradictory, because the ideal induction has shown it to be impossible. But the result is that in the absence of any interfering contradiction every particular proposition is possible in the substantive logical sense, and its contradictory universal proposition is impossible. But where contradiction interferes this is reversed.

In former publications I have given the appellation of *universal* or *particular* to a proposition according as its *first* quantifier is Π or Σ. But the study of substantive logical possibility has led me to substitute the appellations *negative* and *affirmative* in this sense,

and to call a proposition *universal* or *particular* according as its *last* quantifier is Π or Σ. For letting l be any relative, one or other of the two propositions

$$\Pi_i \Sigma_j l_{ij} \qquad \Sigma_i \Pi_j l_{ij}$$

and one or other of the two propositions

$$\Pi_j \Sigma_i l_{ij} \qquad \Sigma_j \Pi_i l_{ij}$$

are true, while the other one of each pair is false. Now, in the absence of any peculiar property of the special relative l, the two similar forms $\Sigma_i \Pi_i l_{ij}$ and $\Sigma_j \Pi_i l_{ij}$ must be equally possible in the substantive logical sense. But these two propositions cannot both be true. Hence, both must be false in the ideal world, in the absence of any constraining contradiction. Accordingly, these ought to be regarded as universal propositions, and their contradictions, $\Pi_i \Sigma_j l_{ij}$ and $\Pi_j \Sigma_i l_{ij}$, as particular propositions.

There are two opposite points of view, each having its logical value, from one of which, of two quantifiers of the same proposition, the preceding is more important than the following, while from the other point of view the reverse is the case. Accordingly, we may say that an affirmative proposition is particular in a secondary way, and that a particular proposition is affirmative in a secondary way.

If an index is not quantified at all, the proposition is, with reference to that index, *singular*. To ascertain whether or not such a proposition is true of the ideal world, it must be shown to depend upon some universal or particular proposition.

If some of the quantifiers refer not to hecceities, having in themselves no general characters except the logical characters of identity, diversity, etc., but refer to *characters*, whether non-relative or relative, these alone are to be considered in determining the "quantity" of an ideal proposition as universal or particular. For anything whatever is true of *some* character, unless that proposition be downright absurd; while nothing is true of *all* characters except what is formally necessary. Consider, for example, a dyadic relation. This is nothing but an aggregation of pairs. Now any two hecceities may in either order form a pair; and any aggregate whatever of such pairs will form *some* dyadic relation. Hence, we may totally disregard the manner in which the hecceities are connected

in determining the possibility of a hypothesis about *some* dyadic relation.

Characters have themselves characters, such as importance, obviousness, complexity, and the like. If some of the quantified indices denote such characters of characters, they will, in reference to a purely ideal world be paramount in determining the quantity of the proposition as universal or particular.

All quantitative comparison depends upon a *correspondence*. A correspondence is a relation which every subject[1] of one collection bears to a subject of another collection, to which no other is in the same relation. That is to say, the relative "corresponds to" has

$$\sum_{u} u \cdot (\imath \Im \breve{u})$$

not merely as its *form*, but as its *definition*. This relative is transitive; for its relative product into itself is

$$[\textstyle\sum_{u} u \cdot (\imath \Im \breve{u})][\sum_{v} v \cdot (\imath \Im \breve{v})] \prec \sum_{u}\sum_{v} uv \cdot (\imath \Im \breve{u})(\imath \Im \breve{v})$$
$$\prec \sum_{u}\sum_{v} uv \cdot (\imath \Im \breve{u} \Im \breve{v}) \prec \sum_{u}\sum_{v} uv \cdot (\imath \Im \overline{u\breve{v}}) \prec \sum_{w} w \cdot (\imath \Im \breve{w})$$

But it is to be observed that if the P's, the Q's, and the R's are three collections, it does not follow because every P corresponds to an R, and every Q corresponds to an R that every object of the aggregate collection P $\dagger$ Q corresponds to an R. The *dictum de omni* in external appearance fails here. For P may be $[u \cdot (\imath \Im \breve{u})]$R and Q may be $[v \cdot (\imath \Im \breve{v})]$R; but the aggregate of these is not $[(u \dagger v) \cdot (\imath \Im \overline{u \dagger v})]$R, which equals $[(u \dagger v) \cdot (\imath \Im \breve{u}) \cdot (\imath \Im \breve{v})]$R. The aggregate of the two first is $\{(u \Im v) \cdot [v \cdot (\imath \Im \breve{v}) \dagger \imath \Im \breve{u}] . [u \cdot (\imath \Im \breve{u}) \dagger \imath \Im \breve{v}]\}$R, which is obviously too broad to be necessarily included under the other expression. Correspondence is, therefore, not a relation between the subjects of one collection and those of another, but between the collections themselves. Let $q_{\alpha i}$ mean that i is a subject of the collection, α, and let $r_{\beta ji}$ mean that j stands in the relation β to k. Then, to say that the collection P corresponds to the collection Q, or, as it is sometimes expressed, that "*for* every

[1] I prefer to speak of a member of a collection as a *subject* of it rather than as an *object* of it; for in this way I bring to mind the fact that the collection is virtually a quality or class-character.

subject of Q there is a subject of P," is to make the assertion expressed by

$$\Sigma_\beta \Pi_i \Sigma_j \Pi_k \bar{q}_{Pi} \dagger r_{\beta j} \cdot (l_{ik} \dagger \bar{r}_{\beta kj}) \cdot q_{Qj}.$$

In the algebra of dual relatives this may be written

$$\Sigma_\beta P \prec \bar{q} \mathbin{;} [\bar{r}_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] \bar{q} Q.$$

The transitivity is evident; for

$$\begin{aligned}
&\Sigma_\beta \Sigma_\gamma \bar{q} \mathbin{;} [r_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] \bar{q} \{\bar{q} \mathbin{;} [r_\gamma \cdot (l \mathbin{;} \bar{r}_\gamma)] \bar{q}\} \\
&\prec \Sigma_\beta \Sigma_\gamma \bar{q} \mathbin{;} [r_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] \{\bar{q} \bar{q} \mathbin{;} [r_\gamma \cdot (l \mathbin{;} \bar{r}_\gamma)] \bar{q}\} \\
&\prec \Sigma_\beta \Sigma_\gamma \bar{q} \mathbin{;} [r_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] \{\mathsf{T} \mathbin{;} [r_\gamma \cdot (l \mathbin{;} \bar{r}_\gamma)] \bar{q}\} \\
&\prec \Sigma_\beta \Sigma_\gamma \bar{q} \mathbin{;} [r_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] [r_\gamma \cdot (l \mathbin{;} \bar{r}_\gamma)] \bar{q} \\
&\prec \Sigma_\beta \Sigma_\gamma \bar{q} \mathbin{;} [r_\beta r_\gamma \cdot (l \mathbin{;} \bar{r}_\beta \mathbin{;} \bar{r}_\gamma)] \bar{q} \\
&\prec \Sigma_\beta \bar{q} \mathbin{;} [r_\beta \cdot (l \mathbin{;} \bar{r}_\beta)] \bar{q}.*
\end{aligned}$$

Not only is the relative of correspondence transitive, but it also possesses what may be called *antithetic transitivity*. Namely, if c be the relative, not only is $cc \prec c$ but also $c \prec c \mathbin{;} c$. To demonstrate this very important proposition is, however, far from easy. The quantifiers of the assertion that for every subject of one character there is a subject of another are $\Sigma_\beta \Pi_i \Sigma_j \Pi_k$. Hence, the proposition is particular and will be true in the ideal world, except in case a positive contradiction is involved.

Let us see how such contradiction can arise. The assertion that for every subject of P there is a subject of Q is

$$\Sigma_\beta \Pi_i \Sigma_j \Pi_k \bar{q}_{Pi} \dagger r_{\beta ij} \cdot (l_{ik} \dagger \bar{r}_{\beta kj}) \cdot q_{Qj}.$$

This cannot vanish if the first aggregant term does not vanish, that is, if $\Pi_i q_{Pi}$ or there is no subject of P. It cannot vanish if everything is a subject of Q. For in that case, the last factor of the latter aggregant disappears, and substituting l for r_β the second aggregant becomes ∞. The expression cannot vanish if every subject of P is a subject of Q. For when l is substituted for r_β, we get

$$\Pi_i \bar{q}_{Pi} \dagger q_{Qj}.$$

If P has but a single individual subject and Q has a subject, for every P there is a Q. For in this case we have only to take for β

*It must be remembered that to a person familiar with the algebra all such series of steps become evident at first glance.

the relation of the subject of P to any one of the subjects of Q. But if P has more than one subject, and Q has but one, the expression above vanishes. For let 1 and 2 be the two subjects of P. Substituting 1 for i, we get

$$\Pi_k r_{\beta 1 j} \cdot (1_{1k} \Psi \breve{r}_{\beta k j}) \cdot q_{Qj}$$

Substituting 2 for i we get

$$\Pi_k r_{\beta 2 j} \cdot (1_{2k} \Psi \breve{r}_{\beta k j}) \cdot q_{Qj}$$

Multiplying these

$$\Pi_k \Pi_{k'} r_{\beta 1 j} \cdot r_{\beta 2 j} \cdot (1_{1k} \Psi \breve{r}_{\beta k j}) \cdot (1_{2k'} \Psi \breve{r}_{\beta k' j}) \cdot q_{Qi}$$

Substituting 2 for k and 1 for k', this gives

$$r_{\beta 1 j} \cdot r_{\beta 2 j} \cdot \breve{r}_{\beta 2 j} \cdot \breve{r}_{\beta 1 j} \cdot q_{Qi}$$

which involves two contradictions.

It is to be remarked that although if every subject of P is a subject of Q, then for every subject of P there is a subject of Q, yet it does not follow that if the subjects of P are a part only of the subjects of Q, that there is then not a subject of P for every subject of Q. For example, numbering 2, 4, 6, etc., as the 1st, 2nd, 3rd, etc., of the even numbers, there is an even number for every whole number, although the even numbers form but a part of the whole numbers.

It is now requisite, in order to prove that $c \prec c \mathbin{;} c$, to draw three propositions from the doctrine of substantive logical possibility. The first is that given any relation, there is a possible relation which differs from the given relation only in excluding any of the pairs we may choose to exclude. Suppose, for instance, that for every subject of P there is a subject of Q, that is that

$$\Sigma_\beta \; \breve{q} \mathrm{P} \prec [r_\beta \cdot (1 \mathbin{;} \breve{r}_\beta)] \breve{q} \mathrm{Q}.$$

The factor $(1 \mathbin{;} \breve{r}_\beta)$ here has the effect of allowing each correlate but one relate. Each relate is, however, allowed any number of correlates. If we exclude all but one of these, the one retained being, if possible, a subject of Q, we have a possible relation, β', such that

$$\Sigma_{\beta'} \; \breve{q} \mathrm{P} \prec [r_{\beta'} \cdot (1 \mathbin{;} \breve{r}_{\beta'}) \cdot (\breve{r}_{\beta'} \mathbin{;} 1)] \breve{q} \mathrm{Q}.$$

The second proposition of substantive logical possibility is that whatever is true of *some* of a class is true of the whole of *some* class. That is, if we accept a proposition of the form $\Sigma_i \, a_i \cdot b_i$, we can write

$$\Sigma_\gamma \, \Pi_i \, \breve{q}_{\gamma i} \Psi \breve{a}_i \Psi b_i,$$

though this will generally fail positively to assert, in itself, what is implied, that the collection j excludes whatever is a but not b, and includes something in common with a. There are, however, cases in which this implication is easily made plain.

Applying these two principles to the relation of correspondence, we get a new statement of the assertion that for every P there is a Q. Namely, if we write a_{ai} to signify that i is a relate of the relative r_a to some correlate, that is if $a_{ai} = (i \prec r_a \Upsilon)$, if we write b_{aj} to signify that j is a correlate of the relative r_a to some relate, that is if $b_{aj} = (j \prec \breve{r}_a \Upsilon)$, and if we write p_{ca} to signify that r_a is an aggregate of the relative r_c, that is, if $p_{ca} = (r_a \prec r_c)$, then the proposition that for every subject of P there is a subject of Q may be put in the form,

$$\Sigma_c \Sigma_\gamma \Pi_x \Pi_y \Sigma_\beta \Sigma_\alpha \Pi_u \Sigma_i \Sigma_j \Pi_\beta \Pi_u \Pi_v$$
$$[\bar{p}_{cu} \Psi a_{ai} \cdot q_{Pi} \cdot b_{aj} \cdot q_{Qj} \cdot q_{\gamma j} \cdot (\bar{a}_{au} \Psi 1_{iu}) \cdot (b_{av} \Psi 1_{jv}) \cdot (\bar{p}_{c\beta} \Psi 1_{\alpha\beta} \Psi \bar{a}_{\beta i} \cdot b_{\beta j})] \cdot (\bar{q}_{Px} \Psi a_{\beta x} \cdot p_{c\beta}) \cdot (\bar{q}_{Qy} \Psi \bar{q}_{\gamma y} \Psi b_{\alpha y} \cdot p_{c\alpha}).$$

This states that there is a collection of pairs, c, any single pair of which, α, has for its sole first subject a subject of P, and for its sole second subject a subject of Q which is at the same time a subject of a collection, j, and that no two pairs of the collection, c, have the same first subject or the same second subject, and that every subject of P is a first subject of some pair of this collection, c, and every subject of Q which is at the same time a subject of γ is a second subject of some pair of the same collection, c.

The third proposition of the doctrine of substantive logical possibility of which we have need is that all hecceities are alike in respect to their capacity for entering into possible pairs. Consequently, all the objects of any collection whatever may be severally and distinctly paired with all the objects of a collection which shall either be wholly contained in, or else shall entirely contain, any other collection whatever. Consequently,

$$\Pi_P \Pi_Q \Sigma_c \Sigma_\beta \Pi_x \Sigma_\alpha \Pi_y \Sigma_\beta \Pi_u \Sigma_i \Sigma_j \Pi_u \Pi_v \Pi_\beta \Pi_m \Pi_n$$
$$[\bar{p}_{cu} \Psi a_{ai} \cdot q_{Pi} \cdot b_{aj} \cdot q_{Qj} \cdot (\bar{a}_{au} \Psi 1_{uu}) \cdot (b_{av} \Psi 1_{vj}) \cdot (\bar{p}_{c\beta} \Psi 1_{\alpha\beta} \Psi \bar{a}_{\beta i} \cdot b_{\beta j})] \cdot (\bar{q}_{fx} \Psi a_{\beta x} \cdot p_{c\beta}) \cdot (\bar{q}_{fy} \Psi b_{\alpha y} \cdot p_{c\alpha}) \cdot (\bar{q}_{\beta m} \Psi q_{Qm} \Psi \bar{q}_{Qn} \Psi q_{fm}).$$

Although the above three propositions belong to a system of doctrine not universally recognised, yet I believe their truth is unquestionable. Suppose, now, that it is not true that for every subject of P there is a subject of Q. Then, in the last formula, $\Pi_m \bar{q}_{Qm} \dotplus q_{Qm} \prec 0$. This leaves for the last factor $\Pi_n \bar{q}_{Qn} \dotplus q_{Qn}$, and then the formula expresses that for every subject of Q there is a subject of P. In other words, we have demonstrated the important proposition that *two collections cannot be disparate in respect to correspondence*, but that for every subject of the one there must be a subject of the other.

The theorem $c \prec c \jmath c$ is now established; for since of any two collections one corresponds to the other, we have $\infty \prec c \dotplus \bar{c}$ or (non-relatively multiplying by $\bar{c}$) $\bar{c} \prec c$. Hence, $c \prec 1 \, c \prec (\bar{c} \jmath c)$ $c \prec \bar{c} \jmath c \, c \prec c \jmath c \, c$; and, by the transitive principle $c \, c \prec c$, we finally obtain $c \prec c \jmath c$.

Thus is established the conception of *multitude*. Namely, if for every subject of P there is a subject of Q, while there is not for every subject of Q a subject of P, the *multitude* of Q is said to be *greater* than that of P. But if for every subject of each collection there is a subject of the other, the *multitudes* of the two collections are said to be *equal* the one to the other. We may create a scale of objects, one for every group of equal collections. Calling these objects *arithms*, the first arithm will belong to 0 considered as a collection, the second to individuals, etc. Calling a collection the counting of which can be completed an *enumerable* collection, the multitude of any enumerable collection equals that of the arithms that precede its arithm. Calling a collection whose multitude equals that of all the arithms of enumerable collection a *denumerable* collection (because its subjects can all be distinguished by ordinal numbers, though the counting of it cannot be completed), the arithms preceding the arithm of denumerable collections form a denumerable collection. More multitudinous collections are greater than the collections of arithms which precede their arithm.

Let there be a denumerable collection, say the cardinal numbers; and let there be two houses. Let there be a collection of

children, each of whom wishes to have those numbers placed in some way into those houses, no two children wishing for the same distribution, but every distribution being wished for by some child. Then, as Dr. George Cantor has proved, the collection of children is greater in multitude than the collection of numbers. Let a collection equal in multitude to that collection of children be called an *abnumeral* collection *of the first dignity*. The real numbers (surd and rational) constitute such a collection.

I now ask, suppose that for every way of placing the subjects of one collection in two houses, there is a way of placing the subjects of another collection in two houses, does it follow that for every subject of the former collection there is a subject of the latter? In order to answer this, I first ask whether the multitude of possible ways of placing the subjects of a collection in two houses can equal the multitude of those subjects. If so, let there be such a multitude of children. Then, each having but one wish, they can among them wish for every possible distribution of themselves among two houses. Then, however they may actually be distributed, some child will be perfectly contented. But ask each child which house he wishes himself to be in, and put every child in the house where he does not want to be. Then, no child would be content. Consequently, it is absurd to suppose that any collection can equal in multitude the possible ways of distributing its subjects in two houses.

Accordingly, the multitude of ways of placing a collection of objects abnumeral of the first dignity into two houses is still greater in multitude than that multitude, and may be called abnumeral of the second dignity. There will be a denumerable succession of such dignities. But there cannot be any multitude of an infinite dignity; for if there were, the multitude of ways of distributing it into two houses would be no greater than itself.[1]

[1] Inasmuch as the above theorem is, as I believe, quite opposed to the opinion prevalent among students of Cantor, and they may suspect that some fallacy lurks in the reasoning about wishes, I shall here give a second proof of a part of the theorem, namely that there is an endless succession of infinite multitudes related to one another as above stated, a relation entirely different, by the way, from those of the orders of infinity used in the calculus. I shall not be able to prove by this

We thus not only answer the question proposed, and show that of two unequal multitudes the multitude of ways of distributing the greater is the greater; but we obtain the entire scale of collectional

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second method, as is proved in the text, that there are no higher multitudes, and in particular no maximum multitude.

The ways of distributing a collection into two houses are equal to the possible combinations of members of that collection (including zero); for these combinations are simply the aggregates of individuals put into either one of the houses in the different modes of distribution. Hence, the proposition is that the combinations of whole numbers are more multitudinous than the whole numbers, that the combinations of combinations of whole numbers are still more multitudinous, the combinations of combinations of combinations again more multitudinous, and so on without end.

I assume the previously proved proposition that of any two collections there is one which can be placed in one-to-one correspondence with a part or the whole of the other. This obviously amounts to saying that the members of any collection can be arranged in a linear series such that of any two different members one comes later in the series than the other.

A part may be equal to the whole; as the even numbers are equal in multitude to all the numbers (since every number has a double distinct from the doubles of all other numbers, and that double is an even number). Hence, it does not follow that because one collection can be placed in one-to-one correspondence to a part of another, it is less than that other, that is, that it cannot also, by a rearrangement, be placed in one-to-one correspondence with the whole. This makes an inconvenience in reasoning which can be overcome in a manner I proceed to describe.

Let a collection be arranged in a linear series. Then, let us speak of a *section* of that series, meaning the aggregate of all the members which are later than (or as late as) one *assignable* member and at the same time earlier than (or as early as) a second *assignable* member. Let us call a series *simple* if it cannot be severed into sections each equal in multitude to the whole. A series not simple itself may be conceivably severed into *simple sections*, or it may be so arranged that it cannot be so severed (for example the series of rational fractions arranged in the order of their magnitudes). But suppose two collections to be each ranged in a linear series, and suppose one of them, A, is in one-to-one correspondence with a part of the other B. If now the latter series, B, can be severed into simple sections, in each of which it is possible to find a member at least as early in the series as any member of that section that is in correspondence with a member of the other collection A, and also a member at least as late in the series as any member of that section that is in correspondence with any member of the other collection, and if it is also possible to find a section of the series, B, equal to the whole series, B, in which it is possible to find a member *later* than any member that is in correspondence with any member of the collection, A, then I say that the collection, B, is greater than the collection, A. This is so obvious that I think the demonstration may be omitted.

Now, imagine two infinite collections, the α's and the β's, of which the β's are the more multitudinous. I propose to prove that the possible combinations of β's are more multitudinous than the possible combinations of α's. For let the pairs of conjugate combinations (meaning by conjugate combinations a pair each of which includes every member of the whole collection which the other excludes) of the β's be arranged in a linear series; and those of the α's in another linear series. Let the order of the pairs in each of the two series be subject to the rule that if of two pairs one contains a combination composed of fewer members than either combination of the other pair, it shall precede the latter in the series. Let the order of the pairs in the series of pairs of combinations of β's be further determined by the rule that where the first rule does not decide, one of two pairs shall precede the other whose smaller combination (this rule not applying where one combinations are equal) contains fewer β's which are in correspondence with α's in one fixed correspondence of all the α's with a part of the β's.

In this fixed correspondence each α has its β, while there is an infinitely greater multitude of β's without α's than with. Let the two series of pairs of combinations

quantity, which we find to consist of two equal parts (that is two parts whose multitudes of grades are equal), the one finite, the other infinite. Corresponding to the multitude of 0 on the finite scale is the abnumeral of 0 dignity, which is the denumerable, on the infinite scale, etc.

So much of the general logical doctrine of quantity has been here given, in order to illustrate the power of the logic of relatives in enabling us to treat with unerring confidence the most difficult conceptions, before which mathematicians have heretofore shrunk appalled.

I had been desirous of examining Professor Schröder's developments concerning individuals and individual pairs; but owing to the length this paper has already reached, I must remit that to some future occasion.

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be so placed in correspondence that every pair of unequal combinations of α's is placed in correspondence with that pair of combinations of β's of which the smaller contains only the β's corresponding in the fixed correspondence to the smaller combination of α's; and let every pair of equal combinations of α's be put into correspondence with a pair of β's of which the smaller contains only the β's belonging in the fixed correspondence to one of the combinations of α's.

Then it is evident that each series will generally consist of an infinite multitude of simple sections. In none of these will the combinations be more multitudinous than those of the β's. In some, the combinations of α's will be equal to those of the β's; but in an infinitely greater multitude of such simple sections and each of these infinitely more multitudinous, the combinations of β's will be infinitely more multitudinous than those of the α's. Hence it is evident that the combinations of the β's will on the whole be infinitely more multitudinous than those of the α's.

That is if the multitude of finite numbers be a, and $2^a = b$, $2^b = c$, $2^c = d$, etc $a < b < c < d <$ etc. *ad infinitum*.

It may be remarked that the *finite* combinations of finite whole numbers form no larger a multitude than the finite whole numbers themselves. But there are nfinite collections of finite whole numbers; and it is these which are infinitely more numerous than those numbers themselves.

SCIENCE AND FAITH.[1]

II. INTRODUCTION TO MAN AS A MEMBER OF SOCIETY. (CONTINUED.)

III. ANIMAL SOCIETIES.

WE HAVE seen that the principal agent employed by evolution in the creation of organisms of increasing complexity is association. Individuals join together in aggregates, preserve their independence for a greater or less period of time, gradually adapt themselves to one another, and end by becoming amalgamated in a single organism. Where there were many individuals there is now but one. Cohesion has given rise to continuity among all the parts, that is to say, to morphological unity.

The kind of association which we are now about to consider is entirely different. Here, the individuals, although still parts of aggregates, are unrestrained and distinct; they come and go; their egos are preserved intact; the bond which unites them is virtual not material. Nevertheless, a large body of philosophers regard the two sorts of association as essentially the same; others, but slightly differing from them, restrict themselves to simple comparison. Some writers have gone so far as to contend that their principles and organisations, rudimentary in animals but as real there as in man, as well as the laws that govern them, are identical. We shall see what this amounts to.

We have already learned that morphology and physiology both tend to reduce the causes which lead animals generally, and the

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[1] Translated from Dr. Topinard's MS. by Thomas J. McCormack. For the two preceding articles of this series see Vol. VI., No. 1 and No. 4.

highest particularly, to form temporary or continued associations, to two: the necessity of satisfying the wants of the organism, the upshot of which is egoism, as a matter of imperative duty; and the need of relations with one's fellows, which culminates in altruism, a product of development from egoism by differentiation.

Struggle for existence, emulation, and competition,—three things which hang together,—are the logical consequences of egoism. The best endowed, those which know best how to take advantage of the opportunities offered, survive and increase. The acutest form of this antagonism is where one animal, to stay his hunger, is forced to devour another. A second widely-spread form is *parasitism*, in which the animal takes up his abode upon or within another and partakes gradually of the latter, according to his needs. Next comes *commensalism*, in which the animal still selects its abode on the surface or in the interior of another, but confines its operations to taking advantage of its situation without doing harm to its host. Example, the little red crab of our common oyster. The following cases are of an allied order: the case of Amphibena, a bird which inhabits ant-hills under sufferance of their proprietors; that of *Elaphis esculapis*, which shares its nook in the thicket with a swarm of hornets; and that of the pilot fish and the remora who keep company with the shark.

Next comes the state of *unilateral mutualism*, in which one species is made use of by another and performs services for the latter but without receiving anything in exchange. The instance of the crocodile and of the bird Trochilus, on the banks of the Nile, is well known. This bird performs two services for the crocodile. It enters its mouth and dispatches there the worms and leeches which trouble the crocodile; it flies rapidly away, giving vent to a peculiar cry when the ichneumon, the enemy of the crocodile, approaches, thus apprising its companion of the ichneumon's presence. In return the crocodile shakes its tail whenever it wishes to close its mouth, thus giving the bird warning. The crocodile in no wise recompenses, but contents itself simply with respecting the person of the little animal. The service rendered is unilateral. But it is easy to understand that by the exercise of extremely little intelli-

gence, if not unconsciously, the crocodile may be led to defend its Trochilus. The same remarks are applicable to birds which associate with certain Ungulata—as Hyas and Ardea with the hippopotamus, Textor with the buffalo of Kaffraria, Buphaga with the elephant of Asia, Ardeola with the elephant of Africa—and which follow them and devour the insects lodged in their thick skins. Interest is the sole impulse of these birds, and in all likelihood it would also be that of the Ungulata in defending them.

The domestication of one species by another is a further instance of unilateral mutualism. A good example of this is that of certain ants who reduce other species to slavery and allow themselves to be fed by them. When man causes domesticated animals to administer to his wants, his pleasures, or his caprice, he supports them in return for their pains, but it is also true that he cruelly slays them when they have ceased to be useful or pleasing to him.

As an example of *bilateral mutualism* we shall cite the case of certain aphids and ants. The aphids secrete an abdominal fluid which distends them; the ants are passionately fond of this secretion, suck the same from the aphids, and finally, in order to keep this precious source of nutrition always at hand, provide them with food; the result being that the aphids are converted into genuine milch cows which are kept and watched in stables. Another example is that of the indicator-bird or honey-guide, and man. The former arrests the latter by his cries and points out to him the location of beehives, by which both then profit. If this partnership were not formed, the one could not obtain the chrysalids of which it is fond, nor the other the honey. Continuing thus, we come to the cases where one animal borrows the services of another temporarily, as is the case with the serpent, who is ferried across a river by a duck, or to the cases where several animals assist one another in crossing streams of water, in lifting a large stone, in moving the trunk of a tree, in constructing a dam, in hunting, or in mutual defence.

The second cause which induces animals to associate together is possibly more powerful—*the need of company*. The struggle for

existence is not so general nor merciless as some extreme disciples of Darwin would maintain. There are frequent lulls. Many species do not have antagonistic wants; the animal is not always possessed of blind hunger; he does not always covet the place of his neighbor; his motives for quarrelling are sometimes extremely slight. The Carnivora are the born enemies of the species that constitute their food, but the Herbivora have only a desire for plants, fruits, roots, barks, etc. Both the one and the other have their moments of necessary repose. Rest is as imperious a want as activity. The Carnivora give most of their time to activity, but the Herbivora spend the greater part in rest. Buffon goes too far, but is in a measure right, when he says: "The animals that live on the fruits of the earth are the only ones that form societies. Abundance is the foundation of social instinct, of that gentleness of manner and peacefulness of life which characterise only those who have no grounds for quarrelling." In fact, a danger which keeps one constantly on the alert, a gloomy climate, a desert country, the necessity of always thinking of the prey which one stands in need of, lead to agitation, to defiance, and to egoism. On the other hand, security, the absence of anxiety, beauty of environment, abundance of food, and rumination, lead to *far niente*, to sympathy, and to love. The animal has no aversion for those who intend him no harm; he approaches, regards his observers with curiosity, and even seems to solicit their caresses. Darwin has described the tameness of wild birds towards man. The latter is shunned only by animals who have learned at their cost to fear him. Man is the greatest enemy of animal societies. Prior to his time, they were unquestionably very numerous. The pastures of Pikermi in the Miocene epoch, the innumerable and multifarious herds of mixed species which the first travellers in Central Africa encountered, are a confirmation of this fact. The societies of buffaloes, of beavers, of chamois, and of numerous other mammals, all dwindled and melted away on his coming. Extensive societies of birds are encountered only in regions sparsely settled by man, as in the northern countries which Dr. Labonne visited. Where man does not slay, he domesticates. The natural troops of the

Andes and of the Himalayas have been replaced by more or less domesticated troops. We assist in the destruction of animal societies.

Whatever be the physiological mechanism by which it is engendered, whether that which I have set forth in a preceding article or some other, it is an undeniable fact that the social sentiment does exist in varying degrees in the majority of animals. All, from the reptiles up, but particularly the birds and the higher mammals, possess the emotional sensibility from which it is derived or which is the consequence of it. Animals associate individually with their fellows or with different species; they exhibit sympathy, and they love, sometimes intensely, sometimes unto death. Every one has witnessed the surprising friendships which frequently spring up between two animals of contradictory characters, even among Carnivora,—friendships which sometimes neutralise the most antagonistic instincts. This sensibility is differentiated in a multitude of ways. Mr. Romanes has followed it up under the heading of "Emotions" in his work on *Animal Intelligence*. It is an admitted fact that in domestication man has only developed qualities which pre-existed in the species. No one will deny but altruism has attained its highest development in the dog, to mention but a single instance.

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In the Fishes we meet with five or six kinds of associations or assemblages, to wit: (1) assemblages between species or between individuals of the same species which should be styled *indifferent*. These are numerous even throughout the entire range of invertebrates, as among the sponges, corals, mollusks, and insects, and depend on conditions of nutrition, of temperature, or shelter, of sandy or rocky bottoms, of calm or agitated environments, according as these conditions suit with the same needs of different species. All that is necessary is that such contiguous species should have little ground for quarelling. (2) Assemblages of the same species, the object of which is hunting in company. Such are the shark and the dog-fish who form shoals in the Channel and pursue the herring; or the carps, who also "live together," we are told, and hunt in company. (3) Associations of the same species for distant voy-

ages. The simple fact that we have to deal here with one species only, like the herring or the sardine, proves that such assemblages are less indifferent than the others. At certain times of the year bands of fishes assemble and travel off either for a change of climate by passing from a cold to a warm region, or in order to find certain kinds of food which abound elsewhere. These bands or shoals frequently comprise a countless number of individuals. Fishes enjoy exceptional facilities for such migrations; they are rapid and easy swimmers, and the currents, too, help them much. (4) Migratory associations, having in view the special end of spawning in remote but favorable localities, to which it is their custom to resort for this purpose. (5) Still another sort of this last kind of association, the object of which is less definite. The salmon is an example of it. Born near the sources of rivers, the salmon descends to the sea, sojourns there seven or eight months, and then again ascends in shoals of from thirty or forty to the place whence he came to perform there the functions of reproduction. Are the fish acquainted with one another under such circumstances? We do not know. At any rate, in certain species they play together.

In the Batrachia and Reptiles one of the conditions of assemblages is greatly weakened. These animals have not the same facility for moving about that fishes have; they creep around on the earth and are frequently very clumsy. Among the terrestrial Reptilia certain crocodiles undertake migrations, but only for short distances, along the banks of rivers. Among the marine Reptilia may be cited the turtles who journey annually to deposit their spawn on distant shores. Indifferent assemblages are frequent, for example, among lizards upon a surface exposed to the sun, or among crocodiles upon the shores of a lake or of a river. Does any durable bond actuate them? Crocodiles thus associating are totally indifferent to one another; no tie whatever results from their union. The lizards, on the other hand, live in perfect harmony and play together; some wander about in little bands, like the Varunus and the Gecko. The blind worm (*Anguis*), the rattlesnake, and *Tropidonotus viperinus* also associate in bands. Marine turtles remain together even after spawning, but seem to take no interest in

one another; they neither engage in mutual attack nor make mutual defence, but swim along together from force of habit. Was it this sort of companionship which led to migration for spawning, or was the contrary the case? A special cause of assemblages, entirely passive in character, may be observed in reptiles. I refer to their hibernation, or periodical torpor, during the long months of winter, where great advantage results from keeping each other warm in holes. Snakes and blind worms (*Anguidæ*) are thus frequently found twined together in solid masses. In 1876, in the forest of Fontainebleau, opposite Thomery, while blasting rock, the workmen came upon a cavity containing three hundred and twelve vipers who had taken up their abode there for the winter.

Birds.—These present all kinds of assemblages save that of hibernation, to-wit, indifferent assemblages; assemblages by pure sociability; assemblages for migratory purposes; assemblages between different species; assemblages for nesting together; and family assemblages.

The kind which gives rise to the largest assemblages is migration. The birds are in this regard even more favorably situated than the fishes; they cut the air with almost vertiginous velocity, changing their climate at will. Some in Europe, for example, descend from the northern countries, as is the case with the duck; others, starting from central regions, fly to the shores of the Mediterranean and Africa. The life of a migratory bird is passed as follows: In the winter in the South it lives according to its habits, either alone, or in groups, or, in exceptional cases, in pairs, dating from the preceding season. In the spring it departs. Reaching its destination, it devotes several months to reproduction, and during the time which is left to it it resumes its usual habits. In autumn, or later, it takes its flight again to the South. Sometimes it departs alone and remains alone during the whole passage, as does the woodcock. Sometimes it departs alone but falls in with companions on its way, which is the case with the quail, who ultimately arrives in flocks of some size, part of which stop in Provence, but the majority of which reach Africa. In some cases the two sexes form distinct groups, which do not join each other until after their ar-

rival, the males being in one flock and the females and their young in another, as is the case with the turkey and the fighting sandpiper (*Philomachus*). Most frequently a signal is given, all the individuals of the same species within a certain region assemble, turn, soar upwards, and depart in a body. Of this kind are the passenger-pigeon (*Ectopistes*), the swallow, the stork, the crane, the crow, the goose and the rook. Some journey only by day, others by night. These flocks vary in number from a few individuals to hundreds, to thousands, and, in one instance of the passenger-pigeon, estimated by Audubon, to 1,100,000. Sometimes isolated individuals or whole flocks of other species join them. In the majority of these societies harmony reigns; in others quarrels and serious combats arise. Save in the turkey, there is no noticeable head or chief of the flock, but frequently, as is the case with the crane and wild duck, there are leaders who take the head of the column and relieve each other by turns. Their flight is confused, in the shape of a triangle, whose vertex cuts the air, or in columns, or in groups. Sometimes the aged males, or the females with their young, or even the young males will fly separately. The few couples which are observed are those which had not separated on departure, or who, on returning, had just begun to mate for the coming season. On their arrival the assemblage or flock may remain intact for some weeks, or for one or two months, but in most cases it breaks up and is dispersed. In sum, they all obey collective habits which have been insensibly formed, consolidated, and converted into a periodical instinct, which the bird obeys. A quail, for example, kept in a warm cage, well fed, and ignorant of everything about him, experiences lively agitation at the time of annual migration, seeks to escape, dashes himself against the bars of his cage, and, as the upshot of his desperate attempts, may drop down dead. It would be useless to add that sedentary societies are transformed most readily of all into migratory societies, and that the spirit of sociability which is habitual with them has also its effects upon the latter.

Sedentary assemblages present many gradations from the indifferent or interested form to that which I have styled assemblages by pure sociability. It is not a temporary and intermittent neces-

sity that is in action here, but commonly a quite pronounced need of playing together, of singing together, of making responses, of abandoning oneself to all manner of pranks and crochets—in other words, of thorough enjoyment through companionship. They are permanent, but during intervals either of rut or of the whole series of reproductive phases. They are made up, according to the season and the species, now entirely of males who have completely or partially abandoned their females, now of males and females followed by their young, who have grown up and are continuing their education under the supervision of both parents or of the mother alone, and again of males, of females, and of offspring who are totally emancipated, the former either paying no regard whatever to each other or still continuing united.

Contrasted with the sociable birds of the preceding category, are the unsociable birds. The following are a few types leading from the latter to the former.

The first type is that of birds who are perfectly egotistical, who live entirely alone or indifferently with others without bestowing on them the least concern or paying them the least attention. Examples are the woodcock, the pheasant, the thrush (*Turdus*), the kingfisher, the cuckoo, and the albatross. The second type is of birds who in general life are egotistical, but possess some traces of family sentiment, and occasionally associate with a few of their fellows for purposes of hunting. The eagle, the vulture, and the falcon are varieties of this type. The third type is of birds who assemble in vast numbers without manifesting any interest at all for one another, but who understand on occasions how to combine their movements for common defence. Examples of this type are several marine birds like the sea-swallows and many stilt-birds. The fourth type is composed of birds which are egotistical, but which form closed and exclusive societies into which no strangers are admitted. An example of this type is the swan, who prefers to live alone rather than to join other groups even when it could be admitted. The fifth type is of birds who form open associations where harmony and happiness reign supreme. These are the immense majority. Such are the passenger-birds, the swallows, the Corvidæ, a large number

of stilt-birds and palmipeds, and the creepers. The parrot is the most advanced representative of this type. Parrots make expeditions like those of the cercopithecoid monkeys, which we shall speak of later, form organisations and station sentinels.

One of the most striking proofs of the spirit of sociability among birds is found in the facility with which many of them associate with individuals of different species but slightly distant from them zoölogically. Here again a gradation appears, running from absolutely indifferent assemblages to the most complicated and harmonious societies. The following are the degrees: (1) unsociable species which chance temporarily holds together but who take no interest in one another; examples of which are the eagle, the buzzard, the vulture, and the kite. (2) Species whose mutual company is agreeable but who do not seek one another, who contract no unions with one another, and derive no advantage from their mutual society; examples of these are the nut-hatch (*Sitta*), the tomtit (*Parus*), the finch (*Fringilla*), the kinglet (*Regulus*), and the creeper (*Certhia*). (3) Species which are egoistic and solitary by nature but which possess qualities that lead other species to gather around them in order to take advantage thereof, and who neither avoid these species nor take any notice of them. Examples are the greenshank and the curlew, who by a peculiar warning cry give the danger-signal to all the inhabitants of a locality. (4) Species which associate together pleasantly, the one having qualities by which the other profits. Examples, the godwit (*Limosa*), a genus of stilt-birds (*Hypsibates*), and the avocet; the first, which is more intelligent and more vigilant, ultimately acquires through these unions a considerable authority over the others. Another example are the unions in the marshes of Hungary between the heron, the ibis, the cormorant, the tern, the goose, and the pelican. (5) Sociable species in all their relations with their own fellows and with stranger species, without there appearing to be any interest on either side, the motive being absolutely the instinct of sociability. These are almost the same as those of the preceding fifth type: the passenger birds, the parrots, the Corvidæ, etc.

The last form of assemblages is for nesting in common. Be-

males abandoned by their males immediately after rut sometimes *lay* their eggs all in one nest, not with a view of sharing the common burden but for the better defence of their eggs. The turkey is an example of this type, the male being the sworn enemy of its eggs. The polygamous females of the ostrich do the same, but for a different purpose. We can recall no example of females abandoned by their males actually *nesting* in common. On the other hand this practice is frequent in the second and third periods when the father participates in it. Examples are the gannet, the cormorant, the petrel, the swift, the chimney-swallow, the rook, the heron, the weaver-bird, the bee-eater, etc. At times a single species, and again different species, associate thus together.

Let us stop and consider a few cases. The gannets, one of those species which in other latitudes help to produce guano, have been described among others by Audubon as they live at the mouth of the St. Lawrence. They arrive from the South in successive flocks of from fifteen to one hundred and take up their abode on the islands there. Here they copulate and construct their nests, two feet apart in parallel rows. If one of the females steals the twigs of its neighbors, the others will all combine against her. When they brood the males hunt for them in the surrounding regions and on occasions will even sit themselves. Later, when the young are able to run about, or fraternise with one another, the nests are trampled upon and the lines effaced. At the end of four months about, all is finished, the young quit the rocks, emigrate, and do not return until the following year. Audubon also describes the nesting places of chimney-swallows, which are the same as tree-swallows, at least prior to the transformation of their instincts. These, too, are migratory birds, and form in their nesting places veritable societies. Audubon has counted fifty nests in the cavity of a sycamore tree and has seen as many as eleven thousand swallows repair nightly to this place in search of shelter. He saw as many as one thousand enter a chimney one evening.

The communal nesting-places of the heron (*Ardea*) of our country are extraordinary from another point of view. A more or less extended group of trees is chosen by them in a swampy country.

Thousands of couples repair thither, each tree supporting from fifteen to one hundred nests, together and at different heights with the nests of other species such as Nycticorax, Ardetta, Phalocrocorax, and Herodias. Nothing is more deafening than the hubbub which these various united species make. The most curious case is that of the weaver-bird (*Ploceus*) and particularly that of Philetærus. Levaillant, the South-African traveller, has counted as many as three hundred and twenty nests or couples on the same tree, and in this instance all of the same species. The nests touch and are covered by a sort of umbrella-like tent fastened in the branches. In these cases the subsequent life of the bird is not prejudiced. The Philetærus when its family is broken up returns to its old life with other and different species. In its social intercourse with these no trace survives of the families which temporarily existed in the previous state.

This leads us to close our remarks on birds by insisting on the facts relative to the varied influence which the family instinct exercises on the social instinct. It is certain that in a general way the species which are most sociable are also the most highly endowed with family qualities. And as examples we might cite the passenger birds, the Corvidæ, and the creepers. But a large number of species with a family turn are quite refractory to any kind of social alliances, as is the case with the Raptores. On the other hand the Gallinaceæ, who are considerably averse to family unions are strongly inclined to sociability whether with their fellow-birds or with other species. I need only recall the case of the wild duck who abandons his females and does not return until the young have grown up, but is yet extremely sociable. Of particular cases I may mention the water-hen, who has a strong family turn but forms neither sedentary nor migratory societies, and particularly the Molothrus, which lives a social life but has so little of the family sentiment as to be given to polygamy and polyandry, which, further, does not form couples, and whose female lays its eggs in the nests of others.

In another point of view, while the sexual instinct forcibly brings the sexes together, and the family instinct brings them to-

gether as a matter of option, on the other hand the sexes are frequently observed to separate in general life and to form distinct groups within the flock or apart therefrom. The young males themselves separate from the young females, who stay a much longer time with their mother. Thus in the pheasant, the young males quit their mother in the autumn, whereas the young females do not leave her until the spring. As to the natural duration of the family, which is fixed by the ability of the young to take care of themselves, we have already seen that it is sometimes abbreviated by the return of the sexual desire in the parents, who drive away their young *nolens volens*. Nevertheless, when there is but one brood a year, or where only the young of the last brood are concerned, there is a distinct tendency on the part of the young to remain longer in the society of their mother, who is then not opposed to their staying, or may be even desirous of it. Such is the origin of the coveys of partridges which pass with us the winter and do not break up until springtime, when rut returns. Coveys of this kind even join others and form multifamiliary societies. In the American ostrich (*Rhea*) this occurs; but the society has here little coherency; the members wander off or pass from one flock to another. In the great bustard several families join and form flocks amounting to several hundred individuals; but in the spring during the period of rut the society breaks up. The only case among birds favorable to the theory that the family is the nucleus of society, is that of the guinea fowl. It has from fifteen to twenty young for which both parents care. At the end of the season six or eight families are joined together, harmony reigns in the bosom of this little society, an old male governs it; and yet they do not know how to render each other mutual assistance in times of danger, but all flee in different directions. We shall conclude on this subject later.

Mammals.—These enlist our whole attention. They present all the forms of assemblages, of a more or less social character, which we have as yet encountered: indifferent, accidental, and temporary, for purposes of migration, for purposes of reproduction, sedentary, between different species, and for purposes of hibernation. Marine mammals, who have the same facilities for speedy

locomotion as fishes, bats which fly like birds, and certain Rodentia and Ungulata, offer examples of association for distant voyages. In the same order of facts, we may recall the short journeys which the marmots and chamois undertake in the winter from regions of snow to the valleys. The seals and the Chiroptera afford examples of distant journeys for reproductive purposes. We shall next say a word regarding assemblages for purposes of hibernation.

We have spoken of snakes and slow-worms (*Anguida*) who enter a state of torpidity during the winter, and who are found entwined in large masses in cavities and holes. Birds fly from the cold with too much facility to have any need of hibernation, and besides they are warm-blooded. In the lower mammals hibernation is pretty common, but only in individuals of solitary habits like the hedgehog, the shrew (*Sorex*), the dormouse (*Myoxus*), the hamster (*Cricetus*), and the harvest-mouse. Hibernation in common is rare, but occurs, for example, in the mole, who has a disposition to burrow in common, in the squirrel where the whole family burrow by the side of one another, but it is notably the case with the marmot. In the higher mammals a trace of hibernation, relating not to society but to family life, is observed in the white polar bear during the period of gestation. The female of the white polar bear digs a hole and, getting into it, causes herself to be covered by snow, staying so covered until spring. In short, hibernation points to nothing as regards the disposition of mammals to form societies.

In the lower mammals, such as the Monotremata, the Edentata, and the Insectivora, social troops are not formed at all. The majority, if not all, live solitary lives, and some are entirely wanting in the family spirit, as is the case with the porcupine ant-eater, the Armadillo, the ant-bear, the pangolin, the sloth, the tanrec, and the shrew, while others are less refractory in this respect, like the duckbill and the hedgehog. The aardvark (*Orycteropus*) is the only one of the Edentata that is met in twos or threes. The mole is the only one of the Insectivora who possesses any social instinct; each has its special burrow, but common corridors exist in which as many as fifteen to twenty individuals dwell.

In the Marsupials the progress is scarcely perceptible. The

majority live alone. Still, in the kangaroo-rat several congregate in a common burrow. In the common kangaroo we meet with indifferent assemblages; these animals graze together in bands numbering as high as eighty individuals, the same ones returning on the morrow either as before or with others as chance decides. Sometimes three or four evince a preference for one another, but no mutual interest. On the slightest occasion each one flees in his own direction without any attempt to join the troop again. And yet the kangaroo exhibits some susceptibility to education in the hands of man. All have heard of the kangaroo boxers.

In the Rodentia the progress is apparent. Some live solitary lives like the dormouse, the hamster, the porcupine, the jerboa, the hare, and the squirrel. The jumping-hare, it is said, lives in large families comprising several couples. In the South American rodent Lagostomus, a dozen families occupy the same burrow, over which a male watches and gives the signal in case of danger. The vole or meadow-mouse is very sociable and sometimes lives in large colonies, the burrows of which communicate with one another and are dug side by side in the same field. The voles, and particularly the lemmings, are celebrated furthermore in northern countries for their enormous emigrations. Their excessive fecundity enables them rapidly to exhaust a country, whereupon they set out in quest of new feeding grounds, in obedience to habits which have persisted for ages and frequently survived their reason for being. Mice and rats, as we know, gather in considerable numbers in localities favorable to their wants. Rats sometimes sleep in a sort of common nest, embracing to keep warm. At night they travel in troops, either in quest of new localities, or to make excursions in the open, all the while observing strict rules of prudence. Rabbits are divided into tribes occupying separate fields; each couple has its own burrow, connected with the others. They go out together in the morning and at night are watched over by an old male who apprises them of danger and urges on the stragglers. The marmots live together and have two kinds of dwellings,—one in summer on elevations, the other in winter in lower places, where they hibernate in common from seven to nine months. The prairie-dogs have

what the Indians call villages. Each has its burrow with well-kept winding pathways between; the lookouts show here and there their heads; they pay one another visits and play together; the habitation of some important personage being the main point about which their wanderings centre. If one of them is wounded or killed, another will quickly drag its body into the nearest burrow while the hunter is reloading his piece. Other and not less celebrated villages were those of the musk-rat and the beaver. The huts of the latter are grouped about a pond; all the members of the community join in cutting and hauling trees, in the constructing or repairing of dams, in digging canals, and in storing provisions. Their works are maintained from generation to generation, and from time to time the excess of the population move off and settle farther away.

The question may be asked with regard to the beaver, whether mutual assistance is the original motive of their living in societies, or whether this mutual assistance is a secondary outgrowth. In the prairie dog everything points to the conclusion that the desire for company is the sole motive. In the multitudinous swarms of lemmings necessity and imitation may account for everything.

The Chiroptera are allies of the Insectivora. They all live in bands which hibernate together and sometimes migrate from one distant isle to another. In France there are famous caves which bats have inhabited from time immemorial and where they have accordingly deposited a thick layer of guano. The interesting point in the history of Chiroptera is this: The females, having been abandoned by the males after rut, gather together in groups of a dozen each in some hole of the cave, where they give birth to their young and rear them in common.

The marine mammals present a similar case, which recalls the practice of communal nesting in birds, and which is complete, complex, and prolonged. We shall speak of them now, although in some respects they are allied to the Ungulata.

The marine mammals are all polygamus, with the exception of the walrus and the dugong, which are monogamous. They all live in herds. The whale is less social, often living a solitary life, yet

sometimes forming herds for the purpose of voyaging or of rut. Some assemblages, the main object of which is play and companionship, are also met with, as among the dolphins. There are also sedentary societies. Thus M. Trouessart speaks of a colony of seals who had taken up their abode in the Bay of the Somme. The five hundred sea-lions at the Golden Gate, near San Francisco, which are protected and fed, form also a sedentary colony. But the interesting groups, although difficult of explanation, are those which have the triple object of voyaging, companionship, and reproduction. Let us essay a sketch of them. These assemblages are composed, according to the season, of complete polygamous families, with a male swimming at their head, of groups of so-called solitary males, of groups of pregnant females, of groups of variously aged young, and of scattered bachelor males. Under what circumstances do these elements separate or come together? Let us abridge the description given of one of these species, the Arctocephalus or sea-bear of the Falkland Islands, by Steller and others.

In November, we are told, the old males arrive at these islands and scatter out on the beach in long files. In December the females arrive, and immediately violent combats are fought for their possession. The young males arrive several months later. At the end of April they all put to sea; in the middle of June the beach is deserted. So far, as I should judge, their conduct has reference solely to rut. The female has one to two young and carries them from eight to ten months, which brings us to the following season. The following, then, is the picture which is drawn for us. Each male has from three to fifteen, thirty, and even as many as forty females, and his entire family may amount to as many as one hundred and twenty individuals, which includes, surely, the young of one year. The beach is divided off into sections ten metres square, each occupied by a different family. The females pass their time in sleeping; the young play together like little dogs; the male is near at hand, and looks on; if the young ones come to blows he comes growling upon the scene, separates them, embraces them, and continues with them their game. If the females behave badly he chastises them; they crawl at his feet, seem to beg his pardon,

and shed copious tears. At times males and females weep together. At a period which is not mentioned the old males separate and go away. A little later all of them quit the beach, each family swimming together. What happens afterwards? Do these families and the various other straggling groups unite and form assemblages comparable to herds or societies?

Among the mammals the Carnivora are the counterpart of the Raptores among birds. They live on flesh, spread terror about them, are ferocious, and reap none but the fruits of egoism. In hard times they devour one another, and, when forced, to it, even eat their females and their young. Nevertheless, some associations are formed among them having in view useful ends. At the head stand the Felidæ. These live alone or in couples, chance alone occasionally inducing some of them to unite for the purposes of chase. The leopard is met in troops of from six to eight. The Canidæ vary. The Colsun of Deccan hunts in packs of from fifty to sixty individuals, the dingo in families. The wild dogs of Constantinople and of Egypt are divided into tribes, each having its headquarters and admitting no stranger. The jackal sometimes hunts alone, sometimes in company. Wolves lead solitary lives in summer and combine in winter into large packs. The blue fox of the poles lives in packs, stations sentinels, but are not less unsympathetic for this reason; they quarrel incessantly and engage in bloody combats. The Viverridæ live solitary lives. A species of the mongoose (*Herpestes*) and the daman (*Hyrax*), of Abyssinia, are often found together, and give an instance of association between different species. The Mustelidæ also live solitary lives; of these the badger is the most egoistic specimen. There is one exception, however, the weasel, which has a developed social instinct. Two or three stories have been told about it in this connexion. A man once carelessly attacked a weasel, who, driven to bay, uttered a war-cry to which twenty weasels responded; these, issuing forth in all directions from their burrows, charged the hunter and forced him to flee covered with wounds. This is solidarity. The Ursidæ live partly solitary lives and partly in small troops. The coatis (*Nasua*) in this respect are of two kinds. One lives a solitary life when not in rut, and the

other lives in troops of from fifteen to twenty individuals, conducted by the oldest. But the harmony in these groups is far from perfect. The otters (*Lutra*), finally, live solitary lives, although in one marine species family life is, as we have already remarked, considerably developed.

The Ungulata are quite differently situated from the Carnivora. They are herbiverous, their food is obtained with a minimal effort and without strife. They pass a part of their time in ruminating with that serenity which every one has noticed. Their life has all the quietness and peace which Buffon regarded as the fundamental condition for developing the social spirit. They all live in small or in large herds, at times temporary but generally permanent, with regard to which the sole problem for us is to discriminate between what is accessory to the family and what is social. Some of them emigrate and their societies are then combined in greater or lesser numbers. Among the latter we will cite the reindeer who annually migrates from regions near the pole and returns there to obtain his favorite lichen in herds which have been known to reach one hundred thousand heads; the antelopes of Central Africa who go in quest of fresh pastures in herds numbering as many as fifty thousand heads; the buffalo who was formerly seen in incalculable numbers. A pioneer's wagon once took eight days to cross an unbroken column of bnffaloes.

In the Solidungula all three kinds of herds occur: family, social, and migratory. The first is simply the permanent polygamous family, such as we have described among the wild ass (*Asinus hemionus*) and the onager, and which, as we have seen, was created as much by the desire of the male to have about him a herd as by the sexual impulse. The second is a union of a larger or smaller number of such families; the number of individuals here amounts to hundreds in the so-called turpans or wild horses of Mongolia, and to thousands among the cimarones of La Plata. In the latter there is no observable leader. In the turpans there is also none; the command is collective and is lodged in the heads of the families. When the herd is attacked, they all form in a circle with the mares and the foals in the centre; their style of defence is methodical. The herd

is not a closed one; if a domesticated horse takes refuge with them he is cordially received. Nevertheless, stallions without females and young males likely to give umbrage to the old males are required to follow on one side. The third kind of herd is formed for purposes of migration, and may be either the one or the other of the two preceding kinds, but particularly the second, created or augmented as the circumstances demand. A fourth kind is also met with among some Solidungula and resembles that which we have so frequently encountered between different species of birds. The zebra is an example of this class. It comprises two species, the dauw and the quagga, of which rival herds, numbering from ten to one hundred individuals, do not mingle. One of these, the quagga, receives into its herds other species, such as the gazelle, the antelope, the gnu, and the ostrich. Is it need of company or utility which gives rise to these associations? As in the birds the most vigilant of this species act as guides, particularly the ostrich who is highly esteemed for his prudence and sharpness of sight.

The ruminants have the same kinds of associations. In the guanacos and vicugnas of the Cordilleras the herd resembles that of the wild ass. It is polygamous during the three periods of rut, gestation, and family life. The male is a chief of a herd, is jealous of the young males as they approach puberty, and is followed by his females and their young with devotion if not servility. In the mouflon two species behave differently. In the Tragelaphus of Africa all live solitary lives; when capable of reproduction the males approach the females in the season of rut, form with them a temporary polygamous herd and then abandon them, each resuming his old habits and the females being left alone with their young. In the musimon of Europe, permanent herds exist in which all ages and sexes are mingled. In the season of rut, the males form polygamous herds, with which they retire aside, whilst the remaining young males and females and the males without females select the oldest among them as their leader. When the season of rut is over, all rejoin the herd and pick out a general leader, the strongest and most esteemed among them. The females are merged in the general body, each having sole charge of her offspring. The males

evince no solicitude for the young, but assume their share of the collective responsibility and interfere in a body in times of danger.

Among the Cervidæ the monogamous reindeer is a type apart. There is a general herd in which all ages and sexes are mingled. Rut arrives; couples are formed which go aside on the approach of parturition, afterwards wander around with their little one until the latter has waxed strong, and then rejoin the troop where the family appears to be prolonged. There is a period, thus, at which the herd is represented solely by the young of both sexes. Outside, a few solitary individuals are found, old males which have been driven from the herd. There are several leaders who relieve each other; for example, in the nightly watch. In the stag (*Cervus*) the old solitary males are found isolated; the adult males are most frequently found forming a little herd apart; and the females with the fawns and the brockets are found united. In the season of rut the males capable of reproduction and the females come together and form a temporary herd whilst the celibates and other abandoned individuals gather in a second distinct herd over which they appoint a temporary chief. After rut, the solitary individuals return to their old ways of life; the most sociable of the males remain with their females for a longer or shorter period of time. In Capreolus this union is intimate and protracted. In the Capridæ the whole breaks up into polygamy at the period of rut. The herd is formed by the females and their young of all ages. As is the general rule, the grouty and ill-natured aged solitaries are expelled from the herd. In the Bovidæ the herd is formed upon the model of the European musimon. The male performs the sexual functions, deserts the female who joins her companions, and then assumes the post of chief of the herd in partnership with the other males, one of them being selected to discharge the principal rôle.

In the antelopes differences are observed. There is the herd of the gazelles numbering from forty to fifty individuals and formed of monogamous families; there is the polygamous troop of the capricorns (*Cervicapra*) in which the old females are utilised as sentinels; there is the temporary troop, during times of rut, of the chamois, and the migratory troops, numbering from ten to fifty

thousand heads, of the springbok. We even meet here with associations among different species.

The Pachyderms are the oldest of the Ungulata. Several are on the eve of disappearing, not only by the hand of man but by the law of evolution which requires that species which no longer conform to present conditions of life shall disappear. There is reason for believing that certain of these species formed anciently numerous societies of which we now possess barely the remnants. They all live in troops of from three to more in the tapir, of from four to twenty in the wild boar (*Sus scrofa*) and Phacochœrus, of from four to ten in the rhinoceros, of from three to four or from fifty to sixty in the hippopotamus, and from four or five to fifty and anciently to two hundred in the elephant. The three individuals to which reference was made in the tapirs appear to bear to one another no family relationship, but are rather an indifferent assemblage, such as we meet with in the kangaroo. In the hippopotamus the groups of three or four may constitute families, but the groups of fifty or sixty are certainly assemblages of families. Among the Suidæ, the twenty individuals which I once counted in the hog (*Sus*) correspond without doubt to a maternal family with the young of several farrows and not to a polygamous family, for the male is not at all sociable nor even disposed to make himself the chief of a herd. Assemblages of several polygamous families are met with among the peccaries of South America, concerning which we read: "they come in numerous herds, the male marching at the head and the females following, with the young in the rear."

The elephant may be seen in herds ranging from five to ten, to fifty, to one hundred and fifty, and formerly in one case, to two hundred. Each herd is a family into which no stranger is admitted. The unfortunate individual who has lost his herd or who has escaped from domestication is taken up by none of them. He is obliged to lead a solitary life. They allow him to approach and drink at the same spring, but they never permit him to mingle in the herd at large; thus he becomes ill-natured. The most prudent and most vigilant is chosen as the chief. Generally it is a male but sometimes it is a female; the chief is deposed when his capacities wane.

He has extensive authority and is always obeyed. He has been seen to station as many as five outposts around the herd to whom he gives his orders and whom he changes. Harmony reigns in this society. The cardinal point is that this herd is really a family, I might add, a large family composed of relatives of all degrees. My reasons for so believing is Tennent's statement that each of these herds can be recognised by special physical characters which are common to all. This is a certain proof of consanguinity.

These lines were already written when my friend M. Louis Rousselet, the author of *L'Inde des Rajahs*, informed me that the males were often found separately in small bands. This would indicate a resemblance to many ruminants like the deer and the big-horn. The males always show a tendency to assemble apart, as do the females with the young. This last division would be the repository so to speak, the centre of the community, its constant fraction.

The Monkeys, from whom we still exclude the anthropoids, offer us numerous examples of the fusion of family and social elements, as well as instances of polygamous troops in which the male is master, and also some cases of solitary monogamous life. Several of them undertake journeys, but they do not form special migratory bands.

The Lemurs may be seen according to circumstances in couples, in small families, or in troops. Thus the Maki by day sleeps rolled up in couples, and by night roams about in troops of thirty or more.

The Monkeys of the New World present all forms. The Nyctipithecus, as we have said, is monogamous, but does not form bands. The Mycetes lives in polygamous families of from three to ten members and has been seen in groups of forty, which points to the association of several families. The Ateles lives in small bands in which besides the young and the females are several males. The Cebus lives in large troops comprising both sexes which other kinds of monkeys sometimes voluntarily join. The Saki, the Callithrix, and the Arctopithecus also live in troops of varying magnitude, some forming but one family and others composed of several. In

both cases there is a chief who in the one is the father and in the other the male in highest esteem. The line of demarcation between the isolated polygamous family and the society is difficult to assign with the defective data now at our command.

As to the monkeys of the Old World no doubt prevails. All live in troops formed of banded families. Examples are the Semnopithecus, the Macacus, the Cercopithecus, and the Cynocephalus. The expeditions of Cercopithecus are well known. The strongest male is the chieftain and directs the movements of the troop, stations sentinels, is the first to advance, climbs trees to reconnoitre, accelerates the movements of the tardy, restrains the precipitate, exacts silence, and by divers grunts and growls issues orders which are both understood and obeyed. They all help one another, cleanse one another, and mutually extract thorns and slivers.

The Cynocephalus is more remarkable still. Brehm, who gained his experience of them in Abyssinia, describes their life in considerable detail. Their troops vary from fifteen to one hundred and fifty individuals, quartered in districts of from a mile to a mile and a half wide not far from a spring. We find together, for example, from twelve to fifteen old males and from twenty to thirty females, the rest being the young of different ages. In the morning, or if it rains, they may be seen in the highest galleries and cavities of the rocks massed together in a body, with the young supporting themselves by preference on their mothers, and the older ones on their fathers. Later, or if the morning is clear, they go in search of their breakfast, lifting the stones, tearing up roots, and gathering fruits. After breakfast they climb up again to the rocks, the males take their seats upon the flat slabs and silently contemplate the landscape while the females watch their infants play and quarrel. Towards evening they repair to the spring, seek their evening meal and then pass the night in an old or in some newly found lodging-place. Brehm describes their offensive and defensive tactics under the direction of a commander-in-chief, their habit of prolonged observation before making a decision, the daring of some of them in their bold dashes to extricate a comrade from danger, and their overawing by attitude and look the dogs of their pursuers

who flee forthwith and take refuge behind their masters. He speaks of their collecting stones at a given point to throw at their enemies, of their even carrying these missiles up trees, and of their aiding one another in rolling the largest of these. Harmony reigns in the bosom of these societies, but between different species as the Gelada and Hamadryas old scores are sometimes settled in free and open-handed combat. M. Mizon has encountered in the neighborhood of the Benue, bands of Cynocephali numbering as many as one thousand who would allow no other monkeys such as the Cercopithecus and Colobus in their domains. The most remarkable instances of co-operation which I know of in the Cynocephali is that which Romanes has reproduced—of a regular combat delivered at the Cape against English soldiers. There was a perfect hail of stones. An old grey-headed male directed the operations of the various squads according to the strategic needs. The English were forced to retreat.

In the Anthropoids our knowledge is far from what we should wish. Like the hippopotamus, rhinoceros, and so many other animals, they are gradually becoming extinct, and their present state gives us no indication of what they anciently were. If they live at this day little in social groups, it is likely because they are not numerous. The following is a summary of our knowledge of them. The gibbon and the chimpanzee love to play and frequently unite and render actual concerts by striking with clubs the branches of hollow trees. The gibbon has been seen in troops of from one hundred to one hundred and fifty. The orang-outang has little social instinct; he lives a solitary life when old, or as a member of a family. Wallace has seen a male or female accompanied with semi-adult young, or three or four infants together, but never two males together. The gorilla has been met by Duchaillu twice in bands of from eight to ten individuals. As to the chimpanzee there is the statement of Schweinfurth based on the accounts of natives that the young associate in small troops. But particularly valuable is the exact affirmation of Livingstone which we have already quoted, that one of these species, the Soko, lives in troops composed of monogamous families.

Conclusions.—The mammals, in the matter of society, do not offer the picture which as the successors of the birds we should naturally have expected. The sentiment which engenders the paternal-maternal and monogamous family in the birds is weakened and has been diverted in the mammals, where in most cases it gives rise to the paternal and polygamous family. Also the social sentiment, which most commonly engenders societies in birds, has been weakened and diverted in the majority of the mammals. As a rule the bird is more altruistic, the mammal more egoistic. In the birds the two sentiments of family and society are quite irregularly distributed in the different orders; in the mammals they form a scale running from zero in the lower orders to a high point in the monkeys. The natural linkage of the orders will perhaps explain these differences: they radiate in the birds, they proceed by steps in the mammals.

The lower mammals, such as the Monotremata, the Edentata, and the Insectivora, are hardly better endowed with regard to family than the reptiles. In the Marsupialia, the Carnivora lead a solitary life while the few herbiverous species that graze together are still in the indifferent period. The Chiroptera form a special group. They seem to crowd into caves, not from any social instinct but because they find there conditions suiting their individual tastes. In the Carnivora, though high up in the scale and in intelligence, there are no societies, properly speaking, but simply temporary assemblages, having as their object attack in common, in which ferocity takes the place of cordiality. In some Rodentia two forms of association are highly developed,—the one for migrations on a large scale, and the other sedentary for mutual help and companionship. In the marine mammals association is developed with the twofold end in view of migration and reproduction, in the latter case in the form of polygamous families. In the Ungulata association is generalised under the triple form of isolated polygamous families, of banded polygamous families, or of associated monogamous families, the first being under the direction of a chief who is necessarily the common father, the two others under the conduct of a single chief chosen from among the fittest, or under that of all

the old males, acting as a single person. In the monkeys the associated polygamous form is general but mingled with less spirit of domination and with more altruism in the male.

Among the birds we have noted (1) associations among different species both for companionship and for mutual service, as frequent here as they are rare among the mammals; (2) large temporary associations for migrations, general as a rule, but rare among the mammals; (3) sedentary and permanent associations, of a cordial, gentle, and ingenuous character, quite different from those ordinarily presented by the mammals. A few orders here and there may be made the subject of parallels. The Raptores among the birds and the Carnivora among the mammals are quite analogous. Egoism, monogamy, family spirit, and no social instinct are their common traits. The owl and the weasel are exceptions; both are sociable. The parrots and the monkeys likewise are counterparts. Clamorous, easily teased, high family spirit and sociability, concerted expeditions,—such are their characters. In certain societies of birds, as the rooks, the swallows, and the crows, there are indications of the formation of a species of tribunals for judging and punishing crimes and misdemeanors committed either within the flock or by strangers. In some mammals and notably in the monkeys, sentinels are said to have been punished for neglect of duty in permitting the troop to be surprised. It is certain that some mammals, like the domestic dog, the cat, and the elephant, have a confused but trustworthy notion of good and bad, of what is permitted and what is forbidden, and of what is just and what is unjust.

* * *

Let us summarise now some of our general conclusions.

1. All assemblages of animals, whatever may be the social form in which they have culminated, began as indifferent assemblages. Vague habits were unconsciously established between a few individuals; these habits were extended to others and even between different species. Pleasure resulted. The habits were confirmed, the pleasure grew. The social spirit was the result, it increased and led to organisations of life in common, often in the

roughest and crudest form, but furnishing the framework within which were developed the customs and characters leading up to those which may be observed in the society of man.

2. At their origin these assemblages, whether they were temporary or prolonged, had no object. Each obeyed his own caprice, the impulses and wants of the moment. Some individuals endowed with the spirit of observation, vigilance, and initiative ventured upon some act which the others imitated. Imitation is a powerful factor in all social and individual phenomena; one must be a physician to appreciate its full potency. M. Tarde has assigned to it an exceptional rôle in the life of man; M. Lebon has described its irresistibleness in the case of crowds. It intervenes incessantly and with more efficacy in animals where routine takes the place of reason. I shall always remember on the eve of the siege of Paris in 1870, the concourse of cattle which were gathered in the Bois de Boulogne. They wandered about dumbfounded. If one should start by any chance in one direction, a second, a third, then ten, a hundred, a thousand would blindly follow. The first pushed on by those behind seemed to be the chief, leading, although unconsciously, the entire troop. Hornaday has given a like description of the buffalo on the prairies. In this manner may be comprehended those astonishing migrations of immense bands of fishes, birds, and of some mammals. Chance crowned by success actuated the first, imitation drew after him the others. The habit once acquired the band was formed over again each year. There are migrations which have persisted for ages, although their original motive has ceased to exist. The instincts acquired are modified, transformed, and adapted to new conditions but with difficulty.

3. The causes of the formations of animal societies are numerous. The first is habit following upon indifference. The second is imitation. What shall we put third? We were prepared, we must avow, after our biological review of the conditions of the problem, to find always in the front rank of the facts, individual interest, egoism, that "categorical imperative" which forces the ego to comply forthwith with the physical exigencies of the organism which it represents. It is not so, and why? Because it is not logic that

determines most of the acts of an animal, but spontaneity. Without doubt, the first impulse of the animal touches his conservation; he flees by reflex action when a danger is presented; he throws himself upon his prey when he is hungry; he gives tooth for tooth when attacked. He avoids the traps which are set for him. But when that first impulse is past, under ordinary conditions, the other natural tendencies quickly regain the upper hand. He gives way to his sensibility, he does not reflect, he does not forestall. Between utility and what is pleasing, between the possible pain of to-morrow and the pleasure of to-day he is not long in hesitating.

The true cause of the formation of more or less sedentary and of permanent societies is that altruism which we have seen to be simply the love of self through others and which subsequently becomes a native sentiment as imperious under certain circumstances as egoism. It is the desire, the pleasure, the need of not being alone, of having companions, of exchanging with them one's impressions, of loving and being loved. There are two kinds of animals, those who in daily satisfying their alimentary needs are obliged to be constantly on the alert, defiant, and ready for combat; and those who having no ordinary ground for conflict give themselves up to the enjoyment of living and are naturally inclined to an existence of peacefulness and pleasure. The first are refractory to the social instinct, their egoism interferes. If they join in assemblages it is from necessity, accidentally and temporarily to hunt their prey. What they form is assemblages and not societies. The second kind, when once on the way, rapidly acquire social habits and progressively gain in altruism what they lost in egoism, coming finally into the possession of a social instinct which in many species is quite powerful. Our meaning is not that individual interest is not manifested in their societies, but that it is secondary there. They live together, they are exposed to the same difficulties of existence, and it is necessary that their action should be mutual and concerted. In the social weaver-bird as it is called (*Philetærus socius*), they have combined for the building of nests and for the rearing of the young side by side with one another; they have arrived, without a thought of the ulterior end, at the construction of

a common umbrella-shaped roof for their nests. The beavers most likely gathered together in social assemblages before they undertook the construction of their great works. The leaders which the majority of constituted societies appoint, the expeditions which parrots and monkeys organise, are the outcome of a common interest; but the societies in question were formed beforehand to satisfy the need of living in company.

In a word, sedentary societies, according to the theory which we present, took their rise in and were developed by the altruistic spirit. Individual interest by itself would never lead to anything consistent. Animals, contrary to certain appearances, as well as to the preconceptions of physiology and to ideas quite widely spread, are more sociable than egoistic. We judge them from our point of view. In this light, they are fierce and brutal; when their immediate material needs speak strongly in them, when their legitimate nervosity intervenes, they are violent, much to be dreaded, and quick in defence. But when these needs are subdued or are easily satisfied they are gentle, kind, and affectionate. The numerous species which man has succeeded in domesticating, from the lizard and the snake up to the elephant, are proofs of this. One must not be guided by particular cases, but must look at the facts in their general bearing. The animal is perhaps superior to man in point of altruism! Animal societies are less polished, but perhaps more humane, all things being equal, than our own.

4. We shall not dwell on the subsidiary causes which concur in the foundation of societies and which we have already discussed or touched upon incidentally,—the need of play and of outwardly venting one's surplus of vitality, the impulse to sing, to be noisy, or to be heard, the need of exercising authority, of being feared and admired, and conversely the need of being assisted, protected, petted, and loved. (See *The Monist*, 1896, p. 551.) We shall confine ourselves to our general conclusion regarding the influence of phenomena of reproduction on societies.

5. In the first period of reproduction everything is opposed to the social spirit. The male and the female flee from their fellows, retire aside, and recognise only themselves. The instinct which

presides at this period is egoistic to excess: the male must possess his female. Before reproduction he beats her when she does not yield with alacrity to his desires; afterwards he continues to beat her to assure himself of her being absolutely his. The solitaries are everywhere the most unsociable and the farthest removed from the family spirit even in those species where the adult males remain with their females. Nevertheless, they are the most ardent in the period of rut. In the second period, of brooding or gestation, when the male and the female have separated, both may enter the group of which they form parts; in the mammals the female never misses doing so. But when they remain together, the preceding situation is protracted, although it is less animal in form; they form a couple by themselves, have common joys, and experience no desire for comrades. In the third period two cases again are presented. When the family deprived of the male is maternal, at times the mother takes refuge in the general social group, seeking its protection, and at times she remains apart with her young who fill her whole existence. When the family, on the other hand, is paternal-maternal, the mother, satisfied with having a protector for herself and her young, has no other desire, while the father also is happy in the task which he fulfils. The happiness and egoism of two, which we observe in the preceding periods, have become the happiness and egoism of three. They are indifferent to everything which is not themselves. Nothing could be more contrary to the social spirit. Towards the end, however, the male gets surfeited with his task, wanders away more and more, and finally rejoins his companions, when his social instinct carries the day over his family instinct. At other times, when the young are definitively emancipated, he keeps on with the habits which he has acquired with his consort: family love disappears, conjugal love is left. They remain together, and the year following, throughout their whole life, they begin over again their romance of love and of family life. It is still the egoism of two individuals. The gain of this egoism is the loss of the social spirit.

So much for the monogamous family. Is it the same with the polygamous family? Let us explain first what is meant by the

word polygamy. It is applied vaguely to the three periods of reproduction and differs from promiscuity, which is sometimes improperly used. Promiscuity is free copulation, each one of the two sexes indulging in the function with equal rights and according to its caprice. It is divided into polygamy for the male and polyandry for the female. Polyandry is rare among animals; the infidelities committed by the female are less rare, but they are not uncommon. Generally the female gives herself absolutely for a whole season, and as a rule gives herself to one only. The male in polygamy does not give himself, he takes the females, and considers himself, so long as he is not satiated, as their master. If he remains polygamous in the second period, it is because he maintains his rights of proprietorship, and if he remains polygamous in the third, it is because he still maintains them by including the infants which are the issues of his females. But polygamy in the first period by no means determines his conduct in the second and third. A male may have an entire harem in the first and yet subsequently attach himself to but one female, discharging the duties of a father only with the infants of the latter, in a word, may be monogamous. Example, the little bustaid or Tetrax. The opposite case is presented by the great bustard or Otis. The male has but one female, but as soon as this one has laid and has begun to brood, he goes in search of another and thus founds several families. In short, the polygamy whose influence we are here examining is not that of the first period, which is mere licentiousness, functional incontinence, as in the turkey and the goat, but that of the third period as in the seal or the elephant.

The conjugal and family ties are looser and consequently, as we have seen, less egotistical and less anti-social, according as they are more removed from monogamy. The more females and infants a family comprises, the more the total store of affection, attention, and protection of which the male is capable is weakened and dispersed. The more this family resembles a harem or a herd of which the male is sultan or chief, the more is it comparable to a little society under the conduct of a single leader. It is very difficult in the accounts of travellers to distinguish the simple numerous family

from the troop or herd of small dimensions. In the Ungulata, the polygamous family often comprises the young of two or three years, although a little later when they have become capable of reproduction their parents usually drive them away. But in other cases, as in the elephant, the young remain in the troop, procreate there, or more probably abandon the troop temporarily to return to it again with their young, with the result that in the end the herd is consanguineous and formerly often embraced as many as a hundred or two hundred members. It is certain that some societies of monkeys are simply augmented families of this kind.

Are polygamous families more capable than monogamous families of forming what Espinas calls *peuplades*, and which we regard as societies *par excellence*? This is the important point to know. Reason answers in the affirmative. Polygamy disperses the sentiment of sympathy, monogamy concentrates it. Polygamy is the egoism frequently of from fifteen to twenty individuals; monogamy is the egoism of three. We have seen numerous instances of polygamous families associating, as in the Tarpan and the buffalo; we have also seen monogamous families, as in the reindeer. But it is my opinion that the former are the most frequent.

We shall take it for granted, then, that polygamy tends more strongly to the formation of animal societies, than monogamy, although it is a lower form of family than the latter. A last reason tells us so. The family of three is a narrowed individuality, intermediary between the individual proper and social collectivity. The family of ten or twenty is a large and diffuse individuality, also intermediary but approaching to collectivity.

It remains to be seen whether, viewing the instrumentality of the young alone, the family favors the formation of society. We have seen, and only the fear of being too prolix has prevented us from dwelling upon it, that the young are invariably controlled by a single dominating tendency—the desire of getting out of their nests as soon as possible, of giving free vent to their activity, and of emancipating themselves, while braving unknown dangers and forgetting their parents. But we have also seen that they are possessed of a powerful impulse to play and to tease one another, to

cry out and to compete in song, even meeting from time to time in some common place for this purpose. To have comrades is a necessity with them. There exist, thus, two contradictory tendencies. The result in the young varies with the species, but in general the more the family state is prolonged the stronger does the habit of living together grow; the more they are conscious of their weakness, the more easily is their food obtained, as in the Herbivora, and the more they yield to the desire of being together; whilst under opposite conditions they abandon themselves readily to their instinct of liberty and of egoism. Nevertheless, small groups of young are formed for hunting in concert among the Carnivora; but occasionally more extensive groups, afterwards rallying to a general flock, are found among the Ungulata.

However a third factor is bound to intervene some day in the case of the young, which puts an end to their inclinations either for independence or for life in common—the arrival of puberty. Birds or mammals, all surrender themselves to the sexual instinct; the solidest ties are broken and the accomplishment of the first act of reproduction takes precedence over everything.

It is certain, however, that the spirit of sociability is most developed in the young who have not yet attained puberty, that it is maintained fairly well after the first rut and even after the first family state, and that it then wanes and quickly drops to zero in the aged males. "Solitaries" are met with in the most sociable species. They are the old males who have spontaneously abandoned life in common or have been expelled from the troop because they were grouty and ill-natured. Age is a factor which must be taken into account, both as regards family and as regards society, when a given species is to be judged. So for the rest there is sometimes wide variations of character, manners, and conduct within the same species. Two travellers may have expressed different opinions and yet both have made correct observations. In many cases it is the mean that has to be sought.

* * *

To adhere faithfully to the plan which we sketched at the outset, whether it be right or wrong, it remains for us to compare

rapidly, not all the forms of association which the vertebrates have presented, but the highest among them, those which best merit the name of societies, with the associations[1] or colonies of lower and intermediary invertebrates.

1. Colonies form a whole, morphologically continuous in all their parts and at all the epochs of their evolution. Societies form diffuse wholes, having a virtual tie only.

2. Colonies tend towards a perfectly definite end, that of multiplying animal forms in time and on the surface of our planet, that of creating new organisms, more and more complex, at the expense of prior simple organisms. This end is wanting entirely in societies. However far solidarity may be carried it is impossible to conceive of a society becoming a new organism or being of any kind. What other end of evolution does it pursue?

As I take it, evolution has no end. It proceeds at random, essays and realises everything that it can, as we have before said, and scoffs at our teleological speculations. Nevertheless, it cannot be denied that among its various operations, regressive, indifferent, and progressive, we are most vividly struck by those which best succeed, by those which engender that admirable harmony revealed by philosophers and lauded by poets. Progressive evolution follows one direction—the *best* by comparison with what has preceded, the best for the species, considering the conditions in which its lot is cast. One of these *best*, as physics and economics have taught us, is the maximum output with a given instrument or organism. We have seen that for the functions of reproduction, progress, amidst attempts of all kinds, has always tended in this direction. Among the fishes we had quantity, but the majority perished; in the higher mammals we had quality, that is, a less number with survival assured. As to the functions of outward life, the same end has been set. Creatures were multiplied in superabundance; what was required was that they should become perfected, that the species should individually yield the maximum output, that is to say,

[1] Evolution has other ways of forming or developing metazoans of increasing degrees of complexity. But the method by organic association is the most widely diffused and the only one which relates to our subject.

that they should exhibit the maximum of activity, of enjoyment, of prosperity, and of well-being. Hence resulted the process of virtual association among demes which evolution follows by habit, and which leads to the strengthening of the ties between the individuals of a species, to their living better, and to the bestowal upon them of more power. By the family, evolution ended in better progeny; by society, it ended in a greater amplitude of life for the species.

The two first differences, in fine, create an abyss between colonies and societies. Comparison seems impossible. But let us continue.

3. In colonies aggregation at first acts by adhesion between individuals which have come from without or are the issue of a common mother; growth is effected by gemmation; total reproduction by the separation of one part, the rest perishing.

In societies aggregation acts by exterior adhesions or by consanguinity; growth by a sort of hypergenesis; reproduction by the separation of a part or swarm, the remainder continuing to live. The parallel is difficult.

4. In colonies division and specialisation of labor are promptly established and more and more accentuated. The individuals form groups which become organs, each concurring within the limits of its specialty in the fulfilment of the general wants. In societies it is the same, each individual is specialised, groups are formed, that is to say, categories; some are favored, others are sacrificed; a hierarchy is established. This is the feature of formal resemblance and one which should be emphasised.

5. In colonies the individuals preserve their independence only for a short time. They almost immediately make concessions to their neighbors, then to groups, and finally to the colony entire; so much so that their individuality becomes entirely absorbed, and they retain no other functions than that of cog-wheels in a great machine. In societies a certain sacrifice of individual independence is also required. The social state is an exchange of concessions; we give in order to receive. But there is a limit; one always preserves the greatest part of one's individuality; one is not bound to

suffer oneself to be absorbed, whatever be the degree of solidarity. This difference is profound.

6. Colonies are presented in the invertebrates in all periods from simple assemblages of individuals with scarcely any adhesion up to complete and absolute solidarisation. We may reduce them hypothetically to three periods. In the first, the individuals still remain their own masters, they lead their own life, and the colonial whole is but their numerical sum. In the second, they have lost half of their individuality, and the colonial whole possesses the other half. In the third, the individuals no longer count as such; they are subordinated to the colonial whole, which wields all the power and all the initiative. In which of these three periods would animal societies fall, on the supposition that we are obliged to class them with colonies, and that we admit they will develop like them in the course of time and in the ascending mammalian scale? In the first, with traces of a tendency here and there towards the second.

In fine, the classing of colonies with societies, which the positivists hold as proper, is a pure fiction, although in some points resemblances exist. If certain laws are applicable to like phenomena in the two orders of association, it is because the grand laws of nature are universal in character and relate as well to sociological or biological facts as to physical, chemical, or astronomical. The plain truth is this: the variously graded associations called colonies are morphological; the associations between demes are virtual. The first create new species, the second perfect them, extend and develop all that they can produce. Will this evolution culminate in the greatest intrinsic good of this or of that species, or in its complete annihilation by very excess of vitality? That is the secret of time. It remains to be learned whether man is situated in this regard the same as the other animals, whether his peculiar attributes do not transform the situation, and whether consequently he will not suggest some modifications of the outlooks gained in the present study.

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THE PHILOSOPHY OF BUDDHISM.

ORIGINAL DUALISM.

Buddhism originated, as all religions do, from the desire to escape the transiency of life with its incidental vicissitudes and to attain the permanent and enduring bliss of an undisturbed existence where there is no pain, no disease, no death, no incertitudes of any kind. As soon as the prevalence of suffering was recognised as an inalienable condition of bodily existence the first attempt at obtaining deliverance from evil was naturally made by a mortification of the body for the sake of benefiting the soul. The body was looked upon as the source of all misery, and a purely spiritual existence was the ideal in which religious men set their hope of salvation. The body is doomed to die, and was therefore considered as an animated corpse. Our material existence is a body of death of which man must rid himself before he can obtain the deathless state. Thus we read in the story of Sumedha, which serves as an introduction to the *Jatakas:*

"Even as a man might rid him of
A horrid corpse bound to his neck,
And then upon his way proceed,
Joyous, and free, and unconstrained;

"So must I likewise rid me of
This body foul, this charnel-house,
And go my way without a care,
Or least regret for things behind.

"As men and women rid them of
Their dung upon the refuse heap,

And go their ways without a care,
Or least regret for what they leave;

"So will I likewise rid me of
This body foul, this charnel-house,
And go my way as if I had
Cast out my filth into the draught."[1]

Sumedha says:

"What misery to be born again!
And have the flesh dissolve at death!

"Subject to birth, old age, disease,
Extinction will I seek to find,
Where no decay is ever known,
Nor death, but all security."[2]

The ideal of Buddhahood, accordingly, was in its original shape the attainment of a purely spiritual condition which it was hoped would afford a perfect emancipation from suffering. It was the same yearning as that of the early Christians, expressed in St. Paul's words:

"O wretched man that I am! who shall deliver me from the body of this death?"

Even Luther, with whom the monistic era of Christianity begins, speaks of his body with the utmost contempt. The term *Madensack*, i. e., a bag full of food for grubs, is a favorite expression of his.

The religious problem, as it presented itself to the ascetic Gautama before he had attained to Buddhahood, was formulated on dualistic principles, but his final solution rested upon a monistic basis. We know little of his philosophical evolution and the phases through which he passed; but the outcome is unequivocal in all important questions that form decisive test-issues as to the character of his system. He was tolerant and showed extreme patience with all kinds of mythologies, even utilising the supersti-

[1] H. C. Warren, in his *Buddhism in Translations*, pp. 7–8. See also the passage quoted from Chapter VI. of the *Visuddhi-Magga*, p. 300.

[2] *Ibid.*, p. 6.

tions of his age to the enhancement of his religion, but he was merciless in his rejection of metaphysicism and dualism.

ANTI-METAPHYSICAL.

After Buddha had surrendered the old dualism, the traditional formulation of philosophical problems lost their meaning; they became what we now call illegitimate questions; and whenever Buddha was confronted with such illegitimate questions, he either refused to answer them or declared openly: "The question is not rightly put."[1] His refusal to answer such questions, which on his plane of thought had become unmeaning and irrelevant, nay, even misleading, can be interpreted as agnosticism, or as a dodge and attempt at straddling, only by those who utterly misconceive the spirit of Buddha's doctrines. When bored with questions by a wandering ascetic, one of those frivolous wranglers who dispute merely for the sake of discussion, Buddha refuses to answer, but when afterwards Ânanda accosts his master he explains why the wandering ascetic received no reply. The reason is here again the error involved in the wrong formulation of the question. Thus if he had replied in the negative, saying that the âtman does not survive death, the wandering ascetic would have said "the Buddha teaches that there is no after-life"; and if he had replied in the affirmative, saying that the âtman survives death, the implication would have been that Buddha believed with the Vedanta philosophers in the existence of an âtman.

Buddha's monism is not materialism; he does not identify soul and body, he only denies the separate existence of soul-entities. There is soul and there is body. There are consciousness-forms and bodily-forms, and both are changing and developing, both are subject to growth and decay. The body is dissolved, and consciousness passes away, yet their forms reappear in new incarnations. There is death and rebirth, and there is continuity of life with its special and individual types. If the soul were identical with the body, it would perish with it; if it were a distinct entity and an immutable âtman, it would not be affected by conduct and

[1] See, for instance, Warren, *Buddhism in Translations*, pp. 167 and 312.

there would be no use in leading a holy life. In either case there is no need of seeking religion. Buddha's solution is, that there are not two things (1) an âtman and (2) the deeds performed by the âtman, but there is one thing—a soul-activity (karma), which operates by a continuous preservation of its deed-forms or samskâras, which are the dispositions produced by the various functions of karma. There is not a being that is born, acts, enjoys itself, suffers and dies and is reborn to die again; but simply birth, action, enjoyment, suffering, and death take place. The life-activity, the deeds, the karma, the modes of motion in all their peculiar forms, alone are real: they are preserved and nothing else. Man's soul consists of the memory-forms, or dispositions, produced by former karmas. There is no self in itself, no separate âtman; the self consists in the deed-forms, and every creature is the result of deeds.

The disciples propose to the Blessed One in the *Samyutta-Nikâya* this question:

"Reverend Sir, what are old age and death? and what is it has old age and death?"

The Blessed One replies:

"The question is not rightly put. O priest, to say: 'What are old age and death? and what is it has old age and death?' and to say: 'Old age and death are one thing, but it is another thing which has old age and death,' is to say the same thing in different ways.

"If, O priest, the dogma obtain that the soul and the body are identical, then there is no religious life; or if, O priest, the dogma obtain that the soul is one thing and the body another, then also there is no religious life. Both these extremes, O priest, have been avoided by the Tathâgata, and it is a middle doctrine he teaches: 'On birth depend old age and death.'" (*Buddhism in Translations*, p. 167.)

PERSONALITY.

But considering the practical importance of personal effort in moral endeavor, how can the denial of the existence of a separate self as the condition of personality be useful in religion?

The answer is, that the denial of the existence of a separate self, an âtman, is not a denial of the real self such as it actually ex-

ists in man's personality. There is no chariot in itself, but there are chariots; there are no persons in themselves, but there are persons. Buddha does not intend to wipe out the personalities of man, but only the false notion of the metaphysical character of personality. Not only did Buddha always endeavor to adapt his teachings to different personalities, but we find generally in Buddhism as much stress laid upon the personal relation of a disciple to the master, as by Luther, who used to say that "it is not enough for a Christian to know that Jesus Christ is the Saviour, he must experience the fact in his heart and must be able to say, 'Jesus Christ has come to save me individually.'"[1]

There is a similar aspiration in Buddhism, which Buddhagosha, in his comments on the Dhammapada, expresses as follows:

"Now when a Supreme Buddha teaches the Doctrine, those in front and those behind, and those beyond a hundred or a thousand worlds, and those even who inhabit the abode of the Sublime Gods, exclaim: 'The Teacher is looking at *me;* The Teacher is teaching the Doctrine to *me.*' To each one it seems as if the Teacher were beholding and addressing him alone. The Buddhas, they say, resemble the moon: as the moon in the midst of the heavens appears to every living being as if over his head, so the Buddhas appear to every one as if standing in front of him." (*Buddhism in Translations*, p. 470.)

Far from being an obliteration of individuality, the denial of the âtman actually involves a liberation of individuality from an error that is liable to stunt all mental growth and hinder man's free development. Buddha takes out of life the vanity of self, which is based upon the dualism of âtman and karma as separate realities. There is no need of bothering about an âtman, but it is important to be mindful, thoughtful, and energetic in all that a man undertakes and does, for the karma is the stuff of which a man is made. One's own personal endeavor and achievements constitute one's personality, and this personality is preserved beyond death, as we read:

"But every deed a man performs
With body, or with voice, or mind,

[1] "Darum ist's nicht genug, dass einer glaubt, es sei Gott, Christus habe gelitten., u. dergl., sondern er muss festiglich glauben, dass Gott ihm zur Seligkeit ein Gott sei, dass Christus für ihn gelitten habe, etc." (Quoted by Köstlin in his *Luther's Theologie.*) Similar passages are frequent in Luther's writings.

'Tis this that he can call his own,
This with him take as he goes hence.
This is what follows after him
And like a shadow ne'er departs."[1]

These lines have reference to the parable of the man whom his family, his friends, and his property leave when he is cited before the judge, while his good deeds alone follow him through the gate of death and plead for him. Speaking without allegory, we ought to say that the deeds, or rather the deed-forms, are the man himself.

There is no duality of a doer and his doings, a thinker and his thoughts, an enjoyer and his enjoyments, a sufferer and his sufferings, an aspirer and his aspirations. There is not an âtman that performs karma; but there is karma which, wherever incarnated in an individual group, appears as an âtman. The words doer, agent, enjoyer, etc., are mere modes of speech. The realities of soul-life consist in doings, thoughts, sufferings, enjoyments, and aspirations. Actions take place, and the peculiar form of every action is preserved as an analogous disposition to repeat that same action in the shape of memory-structures; and all living beings start life as the summed-up memory of their deeds in former existences.

THE DEATHLESS.

There is no âtman-soul; accordingly there is no transmigration of an âtman-soul; yet there is rebirth: there is a reincarnation of the ancestral karma by a preservation and reproduction of the soul-forms transmitted from generation to generation.

Here we must make a distinction between pure forms and materialised forms. By the pure form of a right-angled triangle we mean the mathematical conception in its abstract and absolute distinctness. The relations of the angles and sides are definite conditions of unalterable rigidity. They can be formulated in theories which are readily recognised as eternal verities. The materialist who believes that material bodies alone are real, would say that pure forms are non-existent, but the mathematician knows that a

[1] *Buddhism in Translations*, p. 228.

right-angled triangle is a definite actuality which, whenever an occasion arises, will manifest itself with unfailing exactness. Manifestations of right-angled triangles take place in materialised forms, by which we mean some single drawing made in ink, pencil, or chalk, or a relation obtaining somehow among three points represented by the centres of stars or indicated by rays of light. The actualisation of a pure form may be more or less perfect, but it always exemplifies the laws of pure form and is, so to speak, its incarnation. In this sense Plato speaks of ideas as being above time and space, and Schiller sings of the higher realm of pure forms:

"In den höheren Regionen
Wo die reinen Formen wohnen."

For ethical considerations man must learn to identify himself, not with the materialisation of his thought and aspirations, but with their forms; for the former are transient, the latter eternal. He must let go all attachment to the special and particular embodiment in which his soul appears. He must find his anchorage in that which cannot be destroyed but will last for ever and aye. The pure forms of his soul-being must be understood as possessing him, they shape his brain, the nervous structures of his thoughts, the materialised forms of his sentiments and aspirations; they dominate his life, his energies, his everything, but not *vice versa:* his bodily incarnation does not lord it over the eternal type which in him becomes manifest. The material elements do not possess the directing faculty, for direction is a formal principle.

In this sense Christ existed since eternity as the divine Logos and became flesh in Jesus; and Buddha descended from the Tusita Heaven to earth for the purpose of being incarnated in the son of Mâya. In this same sense Buddhists speak of attaining to the Bôdhi, i. e. enlightenment or Buddhahood, which implies that the Bôdhi existed before Gautama found it. In the same sense, the right-angled triangle and its law existed before Pythagoras; he did not invent the theorem that bears his name: he discovered it. The idea of a right-angled triangle with all its essential relations dawned upon him, became incarnated in him, manifested itself in him.

But here we must pause a moment, for here lies a difficulty which has greatly embarrassed the translators of Buddhist scriptures. The Pâli word *rupa* means "form," but it is frequently used in the sense of materialised form (*rupa kayo*), not only in the sense of pure form; indeed, it must sometimes be translated by body. Thus that which Plato and Schiller would call pure form is in Pâli called *arûpo*,[1] "that which is without *rupa*, the bodiless," commonly translated "the formless."

We read in the Buddhist scriptures that the attainment of Nirvâna is not possible unless we comprehend "the formless," which is the unmaterial, the eternal, the deathless. This deathless, this unmaterial, this "formless," or rather this eternal realm of pure form the *arúpaloca* is not an essence, not an entity, not an individual being or a personal deity; it has no special dwelling, nor is it a locality, or a heavenly abode; and yet it is the most important truth to be known.

"There is, O disciples, something not-born, not-originated, not-made, not-formed. If, O disciples, there were not this not-born, not-originated, not-made, not-formed, there would be no escape for the born, the originated, the made, the formed." *Udâna*, VIII., 3.

The deathless is a mere nothing, if "nothing" means absence of materiality, and yet it is the most important factor of life, for it makes enlightenment possible and is the condition of salvation. In the *Majjhima Nikâya* (Sutta 26), in which Buddha declares that "the deathless has been gained," the theory is set forth that the "Nothing" is not a nonentity, but that it exists; and "of the priest who dwells in the realm of nothingness" it is said that "he has blinded Mâra, made useless the eye of Mâra, gone out of sight of the Wicked One."

He who clings to bodily form, i. e., the materialised incarnation of pure form, and identifies his self with this compound of atoms, this aggregation of material elements, is not free from the illusion of selfhood; he has not found the eternal resting place of

[1] Also spelt âruppo and arûpe. The neuter of arûpo (arúpam) is used as a synonym of Nirvâna.

life; the bliss of Nirvâna, the peace of his soul; he is driven round in a whirl of eternal turmoil, in the samsâra of worldly interests, in aspirations for transient goods.

He who has attained *arûpam*, the formless, surrenders with it all petulancy of self, for jealousy, spite, hatred, pride, envy, concupiscence, vainglory—all these and kindred ambitions—have lost their sense. He is energetic, but without passion; he aspires, but does not cling; he administers, but does not regard himself an owner; he acquires, but does not covet. This is expressed in the *Milindapañha*,[1] where we read:

"Said the king, 'Bhante Nâgasena, what is the difference between one who has passion and one who is free from passion?'

"'Your majesty, the one clings, the other does not cling.'

"'Bhante, what do you mean by "clings" and "does not cling"?'

"'Your majesty, the one covets, the other does not covet.'

"'Bhante, this is the way I look at the matter: both he who has passion and he who is free from passion have the same wish, that his food, whether hard or soft, should be good; neither wishes for what is bad.'

"'Your majesty, he that is not free from passion experiences both the taste of that food, and also passion due to that taste, while he who is free from passion experiences the taste of that food, but no passion due to that taste.'"

THE MIDDLE DOCTRINE.

Buddha calls his solution of the psychological problem the middle doctrine, because it avoids both extremes of what, in the terms of the schoolmen, may be called extreme Realism and extreme Nominalism. Buddha denies that there are things in themselves of any kind. Compounds have no existence outside their parts, and man, like other things, animals, plants, chariots, worlds, etc., is a compound. There is no self in man as a separate entity. Self denotes the whole man. He who says compounds are things in themselves is mistaken, but he who denies the existence of compounds, he who proclaims the doctrine of non-existence is mistaken also. Compounds are real enough, the relation among things and their interaction are not mere illusions. While there are no things

[1] Quoted from Henry Clarke Warren, *Buddhism in Translations*, p. 421. See also *Sacred Books of the East*, XXXV., p. 119.

in themselves, there are forms in themselves. Buddhagosha argues in the *Visudhi-Magga*, Chap. XVIII.:

"Just as the word 'chariot' is but a mode of expression for axle, wheels, chariot-body, pole, and other constituent members, placed in a certain relation to each other, but when we come to examine the members one by one, we discover that in the absolute sense there is no chariot; and just as the word 'house' is but a mode of expression for wood and other constituents of a house, surrounding space in a certain relation, but in the absolute sense there is no house; and just as the word 'fist' is but a mode of expression for the fingers, the thumb, etc., in a certain relation; and the word 'lute' for the body of the lute, strings, etc., 'army' for elephants, horses, etc.; 'city' for fortifications, houses, gates, etc.; 'tree' for trunk, branches, foliage, etc., in a certain relation, but when we come to examine the parts one by one, we discover that in the absolute sense there is no tree; in exactly the same way the words 'living entity' and 'âtman' are but a mode of expression for the presence of the five attachment groups, but when we come to examine the elements of being one by one, we discover that in the absolute sense there is no living entity there to form a basis for such figments as 'I am' or 'I'; in other words, that *in the absolute sense there is only name and form.* The insight of him who perceives this is called knowledge of the truth." (*Ibid.*, p. 133.)

As soon as we abandon the middle doctrine and assume the existence of a self which is supposed to be an entity that is in possession of all the parts of a compound, we must either assume that this entity after the dissolution of its parts will persist or that it will perish; and both views are erroneous because they start from a wrong premise. He who imagines that his self is immortal is mistaken and will cherish foolish ideas as to the mode and place of its future residence. But he who thinks that his self will perish is not less mistaken; he is unnecessarily afraid of death, for there is no self that can perish. Both propositions are senseless, because based on the illusions of either an extreme realism or an extreme nominalism.

He who sees things as they really are ceases to cleave to existence; he does not think that sensation or thought or any one of the aggregates is the âtman, but for that reason his personality is not wiped out.

"He ceases to attach himself to anything in the world, and being free from attachment, he is never agitated, and being never agitated, he attains to Nirvâna in his own person." (*L. c.*, p. 137.)

NOT A DOCTRINE OF ANNIHILATION.

If man is "name and form" and no self in itself, the proposition seems to suggest itself that death ends all; but the doctrine of annihilation is not countenanced by any of the orthodox Buddhists. We read in the *Saṃyutta Nikaya* (XXII., 85):

"Now at that time the following wicked heresy had sprung up in the mind of a priest named Yamaka: 'Thus do I understand the doctrine taught by the Blessed One, that on the dissolution of the body the priest who has lost all depravity is annihilated, perishes, and does not exist after death.'" (*L. c.*, p. 138.)

And a number of priests who had heard the report drew near and said:

"Say not so, brother Yamaka. Do not traduce the Blessed One; for it is not well to traduce the Blessed One. The Blessed One would never say that on the dissolution of the body the saint who has lost all depravity is annihilated, perishes, and does not exist after death." (*Ibid.*)

Then Shâriputra instructs Yamaka by teaching him that there is no such a being as a saint or a man in himself, for all his constituents are transitory and cannot be regarded as his âtman or enduring self; the saint is not bodily form, not sensation, not perception, not any of the predispositions, not consciousness. How then can the saint be annihilated in death? All the constituents of the saint depend upon causation, but holiness and enlightenment are the deathless state which is not touched by death. The *Visuddhi-Magga* comprises this doctrine in these four lines, which sound almost paradoxical:

"Misery only doth exist, none miserable.
No doer is there; naught save the deed is found.
Nirvâna is, but not the man who seeks it.
The Path exists, but not the traveller on it."[1]

And is Nirvâna non-existence? Not at all. It is the attainment of the deathless state, of immateriality, of pure form, of eternal verity, of the immutable and enduring, where there is neither

[1] *L. c.*, p. 146.

birth nor death, neither disease nor old age, neither affliction nor misery, neither temptation nor sin.

"'Wherein does Nirvâna consist?' And to him, whose mind was already averse to passion, the answer came: 'When the fire of lust is extinct, that is Nirvâna; when the fires of hatred and infatuation are extinct, that is Nirvâna; when pride, false belief, and all other passions and torments are extinct, that is Nirvâna." (*L. c.*, p. 59.)

He who attains Nirvâna continues to exist in his personal identity as pure form of a definite character, but he is without any trace of clinging to a particular incarnation. Thus he is no more reincarnated in any special individual, and this is the sense in which Buddha has passed away and yet continues to exist in his bodiless personality, as we read in the *Milindapañha*[1]:

"The king said: 'Is there such a person as the Buddha, Nâgasena?'

"'Yes.'

"'Can he then, Nâgasena, be pointed out as being here and there?'

"'The Blessed One, O king, has passed away by that kind of passing away in which nothing remains which could tend to the formation of another individual. It is not possible to point out the Blessed One as being here or there.'"

THE CONQUEST OF DEATH.

The surrender of the self-illusion with its pretensions brings us practically to the same maxim of life which St. Paul sets forth in 1 Cor., vii., 29–30:

"But this I say, brethren, the time is short: it remaineth, that both they that have wives be as though they had none.

"And they that weep, as though they wept not; and they that rejoice, as though they rejoiced not; and they that buy, as though they possessed not."

This view does not lead to the neglect of the body, but to its being subservient to higher ends and a nobler cause. The Buddha compares the body to a wound which we nurse although we do not love it. Nâgasena says:

"They who have retired from the world take care of their bodies as though they were wounds, without thereby becoming attached to them. (*Buddhism in Translations*, p. 423. Compare *Sacred Books of the East*, XXXV., p. 115.)

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[1] See *Sacred Books of the East*, XXXV., pp. 113–114.

All vicissitudes and afflictions affect the bodily incarnation, not the eternal soul, the pure form or the arúpam, or bodiless, i. e., that which is without rupa; and thus the Saṃyutta Nikâya declares that the saint may be "wretched of body" but can never be "wretched of mind." The actuality of the world, the material reality of existence, the samsâra is absolutely void of permanency. All is transient and nothing endures. Therefore he who sets his heart on anything of the world or its various realisations of form, is sure to suffer; while he who has understood the emptiness of all material existence seeks refuge in the eternal Nirvâna, the domain of eternal verities which, in comparison to bodily realisations, constitute the Void, the Nothing, the existence-less. The eternal verities are immanent in all reality and condition its evolution; they are the aim and purpose of life; they are, to use Goethe's words, "the unattainable of which all actual things are but symbols." They are the nothingness of which we read in the *Majjhima Nikâya* (Sutta 26), that he who dwells in it is "out of the reach of Mâra," the Evil One.

"He has blinded Mâra, made useless the eye of Mâra, gone out of sight of the Wicked One." (*Ib.*, p. 348.)

An ancient Pâli verse (preserved in the *Udâna*, IV., 4) characterises this condition as follows:

"The man whose mind, like to a rock,
Unmoved stands, and shaketh not;
Which no delights can e'er inflame,
Or provocations rouse to wrath—
O, whence can trouble come to him,
Who thus hath nobly trained his mind?"[1]

The belief in self, a separate soul-entity or âtman, is the most serious obstacle to the attainment of the eternal and deathless, because the thought of self infuses all creatures with fear of dissolution as well as a desire for this particular and special copy of its own eternal being. The *Visudhi-Magga* (the Book on the Path of Purity) dwells on the subject in Chapter XXI., where we read:

[1] *Buddhism in Translations*, p. 315.

"To one who considers them [the constituents of being] in the light of their transitoriness, the constituents of being seem *perishable*. To one who considers them in the light of their misery, they seem *frightful*. To one who considers them in the light of their want of an Ego, they seem *empty*.

"He who considers them [the constituents of being] in the light of their transitoriness abounds in faith and obtains the *unconditioned* deliverance; he who considers them in the light of their misery, abounds in tranquillity and obtains the *desireless* deliverance; he who considers them in the light of their want of an Ego, abounds in knowledge and obtains the *empty* deliverance." (*Ib.*, p. 379.)

This is said to explain the stanza:

"Behold how empty is the world,
Mogharâja! In thoughtfulness
Let one remove belief in self
And pass beyond the realm of death.
The king of death can never find
The man who thus the world beholds."[1]

MODERN PSYCHOLOGY.

The world has been greatly astonished in these latter years by the results reached by modern psychologists, Herbart, Fechner, Weber, Wundt, Ribot, etc., who have arrived at the conclusion that there is no soul-being, a theory which received the paradoxical name of "a psychology without a soul." The name is misleading, for the truth is that modern psychology discards the metaphysical conception of the soul only, not the soul itself. The unity of the soul has ceased to be a monad, an atomistic unity, and is recognised as a unification. The personality of a man is a peculiar idiosyncrasy of psychic forms, a system of sensations, impulses, and motor ideas, but it is not a monad, not a distinct entity, not a separate unit. In a word, there is no soul-entity, or soul-substance, or soul-substratum, that is possessed of sensations, impulses, and motor ideas; but all the sensations, impulses, and motor ideas of a man are themselves part and parcel of his soul. Mr. Hegeler expresses it by saying: "I have not ideas, but I am ideas."

The modern theory of the soul is not quite new, for it was clearly outlined by Kant, who counted the notion of a distinct ego-soul as a contradiction, or, as he termed it, one of the paralogisms

[1] *Ib.*, p. 376.

of pure reason. He did not exactly deny the separate existence of an ego, by which he understands apperception as a unit, viz., self-consciousness, but he proved the inconsistency of the assumption and retained the notion only on practical grounds, because he argued that the ego-conception is an idea without which ethics would fall to the ground. Theoretically he rejected the existence of an ego-soul, but for the sake of morality he retained it as a postulate of practical reason.

The ego-soul is nothing but the ancient and famed thing-in-itself in the province of psychology. Metaphysicians of the old school believe that philosophy consists in the search for the thing-in-itself, while the new positivist abandons the idea that there is a separate entity behind or within the parts of things. There is no watch-in-itself; but a peculiar combination of wheels and other mechanical contrivances, together with a dial and the movable hands on the dial, is called a watch. This is as little a denial of the existence of watches as the new psychology is a psychology without a soul. Yet the enemies of the new positivism will still insist that the denial of things-in-themselves implies a philosophical nihilism.

But the new psychology is older still than Kant. As the doctrine of a separate soul prevailed in India among the Brahmans, so the denial of the existence of a separate soul was pronounced more than two thousand years ago by that school of thought which under the leadership of the great Shâkyamuni grew up in opposition to Brahmanism and became known by the name of Buddhism. Not only are the similarities that obtain between modern psychology and Buddhism striking, but we meet also with the same misconceptions and objections. The denial of the existence of a soul-entity is supposed to be a denial of the soul and also of its immortality or its reincarnation.

PROFESSOR OLDENBERG'S VIEW.

Among the expounders of Buddhism Professor Oldenberg of Kiel ranks high. There are others that are his equal, but there is perhaps none who is his superior in scholarship. But with all his philological knowledge, the learned Professor is sadly deficient

in philosophical comprehension. He appears absolutely unable to grasp the significance of the Buddhistic soul-conception, and since his book on Buddha has become a great authority, in Germany almost the sole authority, from which our reading public take their opinions on Buddhism ready-made, his misconceptions have become instilled into the minds of European and American thinkers, and it will be worth while to point out the deficiencies of his propositions.

H. Dharmapâla, the secretary of the Mâha-Bôdhi Society and editor of the Mâha-Bhôdi Journal, the official delegate of Ceylonese Buddhism to the Chicago Parliament of Religions, wrote sorrowfully to me two years ago:

"Professor Oldenberg, the erudite scholar, has not grasped the spirit of the Dharma. He has translated carefully the Pâli words,—and that is all. A philologist may dissect the root of a Pâli word, but it does not make him know the spirit of Buddhism."

I have greatly profited by Professor Oldenberg's researches, which, considered as philological lucubrations, are very valuable, but I have, after all, felt constrained to adopt Mr. Dharmapâla's opinion. I have done so, however, not without hesitation, and not without having previously tried to reach a satisfactory explanation of his position. I shall here briefly call attention to his presentation of the Buddhist soul-conception and then point out the fallacies of his views. Professor Oldenberg says in the chapter entitled "The Soul":

"It is not incorrect to say that Buddhism denies the existence of soul, but this must not be understood in a sense which would in any way give this thought a materialistic stamp. It might be said with equal propriety that Buddhism denies the existence of the body. The body, and in the same sense the soul also, does not exist as distinct and self-sustaining substances, but only as a complex of manifold interconnected processes of origination and decease. Sensations, perceptions, and all those processes which make up the inner life, crowd upon one another in motley variety; in the centre of this changing plurality stands consciousness (viññâṇa), which, if the body be compared to a state, may be spoken of as the ruler of this state."[1] But consciousness is not essentially different from perceptions and sensa-

[1] "The following passage is often repeated in the sacred texts (e. g., in the 'Sâmaññaphala Sutta'): 'This is my body, the material, framed out of the four

tions, the comings and goings of which it at the same time superintends and regulates: it is also a Sankhâra, and like all other Sankhâras, it is changeable and without substance."

Professor Oldenberg adds:

"We must here divest ourselves wholly of all customary modes of thinking. We are accustomed to realise our inner life as a comprehensible factor, only when we are allowed to refer its changing ingredients, every individual feeling, every distinct act of the will, to one and the same identical ego, but this mode of thinking is fundamentally opposed to Buddhism. Here as everywhere it condemns that fixity which we are prone to give to the current of incidents that come and go by conceiving a substance, to or in which they might happen. A seeing, a hearing, a conceiving, above all a suffering, takes place: but an existence, which may be regarded as the seer, the hearer, the sufferer, is not recognised in Buddhist teaching." (*Buddha*. By Dr. Hermann Oldenberg. English Translation, p. 253.)

This is exactly the same as in modern psychology. The assumption of a soul-substance has been found to be a perfectly redundant hypothesis. The soul of man with all its various structures, or, as Buddhists would say, "samskâras," is now conceived as a product of evolution. Life develops the various sense-organs in response to the stimuli of the surrounding world. The function of seeing which is a reaction taking place in response to the impact of the ether-waves of light, results in the appearance of eyes, the function of hearing being a reaction in response to the impact of the air-waves of sound, produces the ear, and the interaction among the senses begets thoughts. The translator of Oldenberg's book, Mr. William Hoey, is not happy in his selection of words, for he says in the passage quoted:

"Sensations, perceptions, and all the processes which make up the inner life, crowd upon one another in motley variety."

Where Oldenberg speaks of *ineinanderströmen* (streaming one into the other), the expression "motley variety" is a redundant addition, and conveys the idea that Buddhistic philosophy regards

elements, begotten by my father and mother . . . , but that is my consciousness, which clings firmly thereto, is joined to it. Like a precious stone, beautiful and valuable, octahedral, well polished, clear and pure, adorned with all perfection, to which a string is attached, blue or yellow, red or white, or a yellowish band."

the soul as a motley crowd of processes. Oldenberg perused the manuscript before it went to press, and it is probable that he took no offence at the expression; indeed the context appears to justify the translator. We would not hold Oldenberg responsible for mistranslations, but English readers know him through the translation only, and for their benefit we feel urged to add a few words in explanation.

Far from regarding the inter-relations of thoughts and sensations as a chance conglomeration, Nâgasena, the famous expositor of Buddhistic philosophy, makes the very opposite statement which in spite of its importance, is nowhere mentioned in Professor Oldenberg's work on Buddha.

We read in the *Milindapañha*:

"It is by a process of evolution that the soul-structures (sankhâras) come to be."

And this statement is inculcated again and again, not less than seven times—a strange anticipation of the evolution theory! And then we read that these soul-faculties that originate through evolution "are not combined indiscriminately" (I. 6, *Sacred Books of the East*, XXXV., p. 87). "First is sight and then thought," for "all that happens happens through natural slope" (p. 90) "because of habit" (pp. 89 and 91) and "on account of an association" (p. 89). In the same sense modern psychologists speak of the "path of least resistance," and the principle of association is so highly appreciated that the English school calls its doctrine the "psychology of association." There is certainly no justification for such a term as "motley variety" in characterising Buddhist psychology. On the contrary, we should be astonished at the anticipations of the most modern ideas.

Those who are accustomed to refer all psychic activity to one and the same identical ego, must, as Professor Oldenberg says, divest themselves of their customary modes of thinking; and he tries hard to do so himself, but he does not succeed.

The new psychology is, in fact, as much simpler than the old one as the Copernican system is simpler than the Ptolemaic sys-

tem, but in order to appreciate this truth we must be acquainted with the facts. The geocentric astronomy appears natural to him who believes that there is an upside and a down, not only on earth, but also in the heavens; and the egocentric psychology is that childlike soul-conception which knows nothing of evolution, but assumes that a stork or other messenger brings into the world at the moment of birth a soul, we do not know whence, which soul is made the lord of the new-born baby with all his inherited tendencies. A certain amount of knowledge is necessary to comprehend the new views in both sciences, but he who has outgrown his mental swaddling clothes will not fail to abandon both the geocentric view in astronomy and the egocentric view in psychology.

VACCHAGOTTA'S QUESTION.

Professor Oldenberg believes that not only the negation of the ego but also the negation of an eternal future must be regarded as the correct solution of the Buddhistic dialectic, and he claims that this was not openly pronounced by the Buddha because he feared to shock the hearts that quailed before the nothing. And yet Oldenberg quotes at the same time the passage of the *Samyuttaka Nikâya* in which the doctrine of annihilation is squarely denounced as a heresy. We read:

"'At this time a monk named Yamaka had adopted the following heretical notion: "I understand the doctrine taught by the Exalted One to be this, that a monk who is free from sin, when his body dissolves, is subject to annihilation, that he passes away, that he does not exist beyond death."'" (Oldenberg, *Buddha*, Engl. ed., p. 281.)

When Sâriputta convinces Yamaka that he does not even in this world appreciate the Perfect One, the monk confesses his error and he says:

"'Such, indeed, was hitherto, friend Sâriputta, the heretical view which I ignorantly entertained. But now when I hear the venerable Sâriputta expound the doctrine, the heretical view has lost its hold of me, and I have learned the doctrine.'" (*Ib.*, p. 282.)

In spite of innumerable passages which prove that Nirvâna is not annihilation, Oldenberg declares that "the doctrine that there

is no ego is equivalent to the proposition: The Nirvâna is annihilation." Professor Oldenberg adds:

"But we can well understand why these thinkers, who were in a position to realise this ultimate consequence and to bear it, abandoned the erection of it as an official dogma of the Buddhist order. There were enough, and more than enough of hopes and wishes, from which he who desired to follow the Sakya's son, had to sever his heart. Why present to the weak the keen edge of the truth: the victor's prize of the delivered is the Nothing? True, it is not permissible to put falsehood in the place of truth, but it is allowable to draw a well-meant veil over the picture of the truth, the sight of which threatens the destruction of the unprepared. What harm did it do? That which was alone of intrinsic worth and essential to excite the struggle for deliverance was maintained in unimpaired force, the certainty that deliverance is to be found only where joys and sorrows of this world have ceased. Was the emancipation of him, who knew how to free himself from everything transitory, not perfect enough? Would it become more perfect if he were driven to acknowledge that beside the transitory there is only the Nothing?" (*Ib.*, 273, 274.)

Buddha, it is true, limited himself to that which conduces to deliverance, holiness, peace, and enlightenment, and gave no answer to questioners who were not prepared to understand his doctrine. Thus Oldenberg quotes the following passage from the *Samyuttaka Nikâya:*

"'Then the wandering monk[1] Vacchagotta went to where the Exalted One was staying. When he had come near him he saluted him. When, saluting him, he had interchanged friendly words with him, he sat down beside him. Sitting beside him the wandering monk Vacchagotta spake to the Exalted One, saying: "How does the matter stand, venerable Gotama, is there the ego (attâ)?"'

"'When he said this, the Exalted One was silent.

"'"How, then, venerable Gotama, is there not the ego?"

"'And still the Exalted One maintained silence. Then the wandering monk Vacchagotta rose from his seat and went away.

"'But the venerable Ânanda, when the wandering monk Vacchagotta had gone to a distance, soon said to the Exalted One: "Wherefore, sire, has the Exalted One not given an answer to the questions put by the wandering monk Vacchagotta?"

"'"If I, Ânanda, when the wandering monk Vacchagotta asked me: "Is there the ego?" had answered: "The ego is," then that, Ânanda, would have confirmed the doctrine of the Samanas and Brahmanas who believe in permanence. If I,

[1] A monk of a non-Buddhistic sect. The dialogue here translated is to be found in the *Samyuttaka Nikâya*, Vol. II., fol. tan.

Ânanda, when the wandering monk Vacchagotta asked me: "Is there not the ego?" had answered: "The ego is not," then that, Ânanda, would have confirmed the doctrine of the Samanas and Brahmanas, who believe in annihilation. If I, Ânanda, when the wandering monk Vacchagotta asked me: "Is there the ego?" had answered: "The ego is," would that have served my end, Ânanda, by producing in him the knowledge: all existences (dhamma) are non-ego?'

"'That it would not, sire.'

"'But if I, Ânanda, when the wandering monk Vacchagotta asked me: "Is there not the ego?" had answered: "The ego is not," then that, Ânanda, would only have caused the wandering monk Vacchagotta to be thrown from one bewilderment into another: "My ego, did it not exist before? but now it exists no longer."'"

Oldenberg's interpretation of this passage is as follows:

"We see: the person who has framed this dialogue has in his thought very nearly approached the consequence which leads to the negation of the ego. It may almost be said that, though probably he did not wish to express this consequence with overt consciousness, yet he has in fact expressed it. If Buddha avoids the negation of the existence of the ego, he does so in order not to shock a weak-minded hearer." (*Ib.*, 272, 273.)

Any one who understands the doctrine of modern psychology will appreciate Buddha's silence, which is amply explained by Buddha's words. Buddha refuses to answer the questions of Vacchagotta, but he gives a satisfactory explanation to Ânanda.

It appears that Vacchagotta was a man who exhibited a hopeless confusion concerning the fundamental notions of the Buddhist psychology. Buddha, it is true, denied the existence of an ego-soul; he denied that that something in man which says "I" can be regarded as a metaphysical soul-principle lording it over all the faculties of man; but Buddha does not deny the reality of man's actual soul, his sensations and motor ideas; he does not deny the presence of consciousness, nor that there is a psychic structure in him that says "I." On the other hand, he does not teach that the soul of man (his sankhâras) will be annihilated in death. He teaches reincarnation, man's soul-structures will reappear, or rather they continue to exist after death. They are impressed upon others, and there is no annihilation; they are preserved exactly in the way in which they manifested themselves. Thus Vacchagotta's question

could not be answered with a straightforward Yes or No. A simple Yes or No would under all conditions simply have increased the questioner's confusion. The question could be answered only after a discussion and complete explanation of the meaning of the term ego, which for reasons not mentioned in the dialogue the Buddha did not see fit to make. Probably he deemed it a waste of time to have a controversy with a professional controversialist and therefore refused to accept his challenge.

Suppose a carpenter's apprentice without education who understood nothing of mathematics, had approached the late Professor Gauss of Göttingen and asked him: "I understand that the Professor denies the reality of circles and lines, that he declares they are purely mental, ideal products of imagination, and quite unsubstantial? Will not the learned Professor answer my question squarely and in a straightforward manner, without reserve and without shirking the issue, Is mathematics substantial or is it not substantial?" What would Professor Gauss have said? Had he said, "mathematical figures are substantial," the apprentice would have acquired an erroneous notion regarding the nature of mathematics; but had the Professor said "Mathematics are unsubstantial and purely ideal," the young fellow would have thought that mathematical constructions were arbitrary and imaginary like dreams. Professor Gauss would probably not have answered the question at all, for whatever he might have said, it would have been bewildering to the questioner. Now, should we say, on reading the report of such an interview, that Professor Gauss had practically taught the non-existence of mathematics? And could we presume that we understood why he avoided to draw the last conclusion of his doctrine; namely, for the reason that he did not want to shock the weak-minded hearer who still clung to the idea that there is a substance of mathematics?

Professor Oldenberg's interpretation of the passage quoted from the *Samyuttaka Nikâya* would make of the Buddha a hypocrite or a coward, for it represents him as not willing to concede the last consequence of his doctrine and without directly telling a lie as trying to make a false impression upon his interviewer. If Vac-

chagotta had been one of Buddha's followers, there might have been a reason for Buddha's not shocking his religious faith, but Vacchagotta belonged to a non-Buddhistic sect, and his question was not made in anxiety or with quivering lips. The context of the passage refutes Professor Oldenberg's interpretation.

Why not understand the passage as it reads? Had the Buddha said "the ego is not," Vacchagotta would imagine that the Buddha believed in annihilation, a doctrine which is unequivocally condemned in the Buddhist canon as a heresy. According to Professor Oldenberg, however, this would be the true import of the Buddhist religion. Vacchagotta, relying on the fact that his ego-consciousness was real, would say: "Did not my ego exist before? and now I am told that there is no ego." In the same way the hypothetical carpenter's apprentice in his interview with Professor Gauss would have said: "The lines which I use in measuring beams and boards are real; and yet this man who is supposed to be a great authority in mathematics tells me that mathematical lines are purely ideal!" We cannot help thinking that if Professor Oldenberg had asked the Buddha whether or not he taught the immortality of the ego, the Buddha would have given him the same answer as he did Vacchagotta: he would have remained silent.

Professor Oldenberg takes a denial of the existence of the ego-soul as a denial of the existence of the soul itself, in the same way that the carpenter's apprentice might have understood that Professor Gauss, not believing in a mathematical substance, denied the existence of mathematics altogether. Truly, to understand Buddhism, we must have an inkling of the fundamental notions of philosophy, and with all due respect for Professor Oldenberg's philological erudition, we cannot help saying that philosophical comprehension is a weakness of his which renders him unable to grasp the meaning of Buddhism.

The soul, according to Buddhism, does not consist of substance but consists of sankhâras, which are sentient structures or forms produced by deeds, by karma, or function. A man's personality is name and form. The name may be preserved and the form may

reappear in new generations. The individual dies, but its form continues by rebirth. There is no individuality in the sense of the Brahmanical âtman theory, but the individuality of a man, his name and form are for that reason real enough; and name and form are either singly, or sometimes together, preserved and reindividualised. There is a continuity in life in which the same form is preserved, and this continuous preservation of form is all that is and can be meant by sameness of personality. This is the secret (if there be any secret about it) of the Buddhist psychology.

IS NIRVÂNA ANNIHILATION?

Professor Oldenberg's conception of Buddhism differs from mine; he says in a letter to me:

"Buddhism, in my opinion, suffers from the contradiction, historically quite conceivable, that on the one hand, it retains the old, concrete, and popular conception of a transmigration of the soul, on the other hand dissolves in its philosophy the idea of a soul as a substratum, an ego-being. This is a contradiction which will never be overcome by your attempt at sublimating the category of karma. Had Buddha not believed in a transmigration of the soul, suicide should have appeared to him as the quickest and best adapted means of making an end of suffering. A few drops of prussic acid would be a better, and at any rate a more rapid remedy than the holy eightfold path."

If this opinion of the learned Pâli Professor be tenable, the Buddha, who is generally regarded as one of the keenest thinkers that ever lived on earth, would have both denied the existence of a thing and at the same time have taught that it migrated from place to place. And we are requested to believe that the Buddha should have been guilty of such a gross contradiction! No, I would rather run the risk of doubting the infallibility of a German professor!

While Professor Oldenberg's summary solution is *prima facie* improbable, it is at the same time based upon incorrectly-stated facts. Buddhism teaches reincarnation, but it does not teach the migration of the soul. Professor Oldenberg's book, although good in many respects, is very deficient in its exposition of the Buddhist psychology, which is just the most important part of Buddhism.

Oldenberg must have overlooked the passages in which the theory of soul-migration, in the sense of an ego-soul migrating from one body into another, is rejected. Buddhism denies that the soul is a substance, and in spite of Professor Oldenberg's statement to the contrary, it denies also most emphatically and unequivocally that there can be any transmigration or transportation of soul-substance. Yet Buddhism asserts the reappearance of the same soul-forms. We read in the *Questions of King Milinda*, III., 5, (*Sacred Books of the East*, XXXV., p. 111):

"Where there is no transmigration, Nâgasena, can there be rebirth?"

"Yes, there can."

"But how can that be? Give me an illustration."

"Suppose a man, O king, were to light a lamp from another lamp, can it be said that the one transmigrates from, or to, the other?"

"Certainly not."

"Just so, great king, is rebirth without transmigration."

"Give me a further illustration."

"Do you recollect, great king, having learnt, when you were a boy, some verse or other from your teacher?"

"Yes, I recollect that."

"Well, then, did that verse transmigrate from your teacher?"

"Certainly not."

"Just so, great king, is rebirth without transmigration."

"Very good, Nâgasena!"

In the Jataka tales and other popular legends expressions are frequently retained which suggest the old Brahmanical conception of a transmigration of soul, but philosophical expositions of the problem leave no doubt about the meaning of the Buddhistic idea of rebirth. At any rate, here is a plain statement in one of the most famous and authoritative Buddhist scriptures, which denies that there is any transmigration of a soul-entity; and thus Professor Oldenberg's charge of inconsistency falls to the ground, as it rests on a misstatement of the Buddhist faith.

Here is another example, adduced by Nâgasena in the *Milinda-panha*:

The mango that is planted rots away in the ground, but it is reborn in the mangoes of the tree that grows from its seed. He

who steals the fruit steals the property of him who sowed the mango. There is no transmigration of a mango-soul from the seed to the fruit, but there is a reconstruction of its form. Thus (as said he who came from Nazareth) the body of a man can be broken down like a temple that is destroyed, but it can and will be built up again. The life of a man does not end with death, for his soul is reincarnated again and again.

And how does this transfer of soul take place? Partly by heredity as is explained by Nâgasena in the illustration of the mango seed, partly by communication. A particular man is not a discrete individual, but a trysting-place of soul-activities, of sankhâras, which are impressed into him by example and education. Thus, a boy in school learns a verse by heart; there is no transfer of soul-substance migrating from the teacher to the pupil, but there is a reincarnation of a certain soul-form. The teacher's words are impressed into the boy; and this is called by Nâgasena "rebirth without transmigration."

Similar passages and similes in explanation of the same idea are found in the *Visudhi-Magga*, where the transfer of soul is illustrated by the reappearance of the form of a face in the mirror, of a voice in its echo, of a seal in its imprint, etc.

Professor Oldenberg knows very well that Nirvâna in the Buddhist texts is not annihilation, but deliverance from evil; and there are innumerable passages which characterise it as the state of highest bliss. Professor Oldenberg quotes several passages from various sources, which corroborate the positive conception of Nirvâna. He says:

"Buddhist proverbs attribute in innumerable passages the possession of Nirvâna to the saint, who still treads the earth:

"'The disciple who has put off lust and desire, rich in wisdom, has here on earth attained the deliverance from death, the rest, the Nirvâna, the eternal state.' Suttasangaha, fol. cû., a Brahmanical ascetic addresses to Sâriputta this question: 'Nirvâna, Nirvâna, so they say, friend Sâriputta. But what is the Nirvâna, friend?' 'The subjugation of desire, the subjugation of hatred, the subjugation of perplexity; this, O friend, is called Nirvâna.'" (*L. c.*, p. 264.)

But Nirvâna may be the *summum bonum*, because it involves the

cutting off of the cause of existence, and the state of Nirvâna may become an actual annihilation at the moment of death. Yet even the final goal of saintship is not characterised as an absolute extinction. Professor Oldenberg quotes the following passages from the *Udâna* (fol. ghau):

"'There is, O disciples, a state, where there is neither earth nor water, neither light nor air, neither infinity of space, nor infinity of reason, nor absolute void, nor the co-extinction of perception and non-perception, neither this world nor that world, both sun and moon. That, O disciples, I term neither coming nor going nor standing, neither death nor birth. It is without basis, without procession, without cessation: that is the end of sorrow.

"'There is, O disciples, an unborn, unoriginated, uncreated, unformed. Were there not, O disciples, this unborn, unoriginated, uncreated, unformed, there would be no possible exit from the world of the born, originated, created, formed.'"

Professor Oldenberg adds the following comments:

"These words seem to sound as if we heard Brahmanical philosophers talking of the Brahma, the unborn, intransient which is neither great nor small, the name of which is 'No, No,' for no word can exhaust its being. Yet these expressions, when viewed in the connexion of Buddhist thought, convey something wholly different. To the Brahman the uncreated is so veritable a reality, that the reality of the created pales before it; the created derives its being and life solely from the uncreated. For the Buddhist the words 'there is an uncreated' merely signify that the created can free himself from the curse of being created (in the 'Dhammapada' it is said, v. 383): 'If thou hast learned the destruction of the sankhâra, thou knowest the uncreated'—there is a path from the world of the created out into dark endlessness. Does the path lead into a new existence? Does it lead into the Nothing? The Buddhist creed rests in delicate equipoise between the two. The longing of the heart that craves the eternal has not nothing, and yet the thought has not a something, which it might firmly grasp. Farther off the idea of the endless, the eternal could not withdraw itself from belief than it has done here, where, like a gentle flutter on the point of merging in the Nothing, it threatens to evade the gaze." (*Ib.*, p. 283, 284.)

Is there no other interpretation of the quoted passages than the one offered by Professor Oldenberg, viz., that the Buddhist faith is equivocal, and that it leaves the question undecided, either as an "unfathomable mystery," or as "resting in a delicate equipoise between the idea of a new existence and nothing"?

It would be difficult here for any man to speak authoritatively,

but it appears to me the solution is not far to seek. The attainment of Nirvâna consists in enlightenment, that is to say, in a recognition of the fundamental truths of religion, which in their practical application are expressed in the noble eightfold path of righteousness. All individual craving has disappeared in the saint; he has become an incarnation of truth, not of theoretical or purely scientific notions concerning the nature of things, but of practical truth which manifests itself in a moral life. Thus Nirvâna is actually an utter annihilation of the thought of self and an embodiment of universal love and righteousness. Those eternal conditions which constitute righteousness are realised in a human heart.

If we translate Buddhist thought into Christian terms, we would say that the attainment of Nirvâna means God-incarnation, and the Buddha is the God-man. Shall we say that the eternal conditions of righteousness are a mere nothing, because they are unsubstantial? Are they non-existent because they are not concrete things, not material objects? That would certainly lead to a serious misconception of the most important facts of existence!

Further, must God be considered as a non-entity when we learn to understand that God is not an individual being? Dwindles the Christian idea of Heaven away, because astronomy finds no place for it in the stars? There are things spiritual the existence of which does not depend upon a definite locality. The Pythagorean theorem is true, and would remain true, even if the world existed no longer. It is an eternal verity and not a mere nothing. This is illustrated in the "Questions of King Milinda" as follows:

"'The king said: 'Venerable Nâgasena, where does wisdom dwell?'

"'Nowhere, O king.'

"'Then, sir, there is no such thing as wisdom.'

"'Where does the wind dwell, O king?'

"'Not anywhere, sir.'

"'So there is no such thing as wind.'

"'Well answered, Nâgasena.'"

It may be difficult to the untrained to understand the paramount importance of eternal verities, but no one can deny their actual presence in life. What other meaning can there be in the

words of Christ when he says: "Heaven and earth may pass away, but my words shall not pass away." The Buddha utters the same sentiment. He says:

"The Buddhas are beings whose word cannot fail; there is no deviation from truth in their speech," etc. (*Buddhist Birth Stories*, p. 18.)

The words of Buddha are not merely the sankhâras of his individual existence, but the eternal verities which shall not pass away, and he who realises them in his soul has attained Nirvâna.

Now, I can see Professor Oldenberg smile, and hear him say, "That is what I mean; Nirvâna is, according to Buddha, the attainment of the eternal verities, and nothing else; accordingly it is tantamount to extinction. Nirvâna is not a place, and the Buddha after his death is no longer a definite individuality that can be pointed out to be here or there. *Ergo* he is dissolved into nothing." To be identical with verities that are eternal but have no dwelling place in space is, in the opinion of many, an annihilation; for ubiquity and nullibility are to their minds two expressions of one and the same thing. Kepler's soul has become the recognition of the three famous laws that bear his name; Ludolf is identified with the calculation of π; Newton with the formulation of the law of gravitation. They attained, each one in his own way, some special aspect of the uncreated, the eternal, the unborn. In the same way the Buddha (in the Buddhistic conception) has become the moral law which is, ever was, and shall remain forever the path of delivery from evil. Immortality is claimed for the Keplers, the Ludolfs, and Newtons, not for their names alone, because their names might be forgotten, but for their souls, for their ideas, for the verities with which they have become identical; and in the same sense, only in the broader field of religious truth, Buddhists believe in the eternal omnipresence of the Buddha. If that be nothing, then "Nothing" stands for the highest and noblest that can be thought of, and Nothing would be the divinest thing in the universe. Indeed, those invisible realities which, when recognised, are called truths, are of greater importance than concrete things and individual beings.

This is plain to every one who understands that truths are real, even though they are not substances or entities. And the same is true of the soul. To deny that volition, cognition and other mental activities are substances, or entities, or that they need a substratum or metaphysical subject in order to be real, is not a denial of their existence—it is simply the consistent consequence of the commonly acknowledged truth that they are not material.

Here lies the main difficulty in understanding Buddhism, which, whether we praise it or condemn it, must be recognised as the most philosophical of all religions. There is no use in understanding the words of the Buddhist texts, if we have no comprehension of their meaning. And how gross Professor Oldenberg's conception is, appears from his proposition that unless Buddha had been guilty of the inconsistency of believing in soul-transmigration, suicide would have been a better remedy for the evils of existence than the noble eightfold path of righteousness.

Suicide causes the dissolution of the individual; it sets an example which in the hearts of others will, according to circumstance, bear evil fruit; it causes consternation and unrest, and can therefore not lead to the cessation of suffering; under no condition could it conduce to the attainment of Nirvâna. He who imagines that but for the supposition of a transmigration of soul, suicide would be a more appropriate and safer method of reaching Nirvâna than the eightfold path of righteousness, has no inkling of the significance of Nirvâna.

Whatever error I may be guilty of in my own representations of Buddhism, be it in essays that I have written or in the *Gospel of Buddha*, this much is sure, that Professor Oldenberg has misunderstood its most salient doctrines, those on the nature of the soul and of Nirvâna. Being a professor who has studied the southern canon of Buddhism in its original documents, he is by many people looked upon as the greatest living authority on the subject, and he can therefore not fail to propagate his misconceptions. Misconceptions in all fields of thought are unavoidable, but if they originate in men who are called upon to be the channels of our information the result will be sad.

Professor Oldenberg is a good scholar, and, I repeat, I gladly acknowledge my indebtedness to him as a philologist; he may also be a good historian, but he has shown himself to be incompetent as an interpreter of Buddhism. His expositions remind us of the parable of the hardwood,[1] that is related in the *Majjhima-nikâyo*, where we read:

"It is exactly, O monks, as if a man who demands hardwood, seeks for hardwood, and looks out for hardwood, climbs over the hardwood of a big hardwood tree, over the greenwood, over the bark, to the boughs and cuts off a twig, taking it along with the idea 'that is hardwood.' Suppose that a clear-sighted man observes him, saying: 'This good man really knows neither hardwood, nor greenwood, nor bark, nor boughs, nor foliage, therefore this good man who demands hardwood, seeks for hardwood, looks out for hardwood, climbs straightway over the hardwood of a large hardwood tree, over the greenwood, over the bark, and cuts off a twig in the opinion that it is hardwood. But the hardwood which he will get from the hardwood of the twig will not serve his purpose.'"

Professor Oldenberg has devoted his life to the decipherment of Sanskrit and Pâli, but he has failed to comprehend the significance of Buddhism. He has climbed over the hardwood of the doctrine of the Buddha without comprehending either its import or possible usefulness, and, presenting us with the foliage of externalities, assures us that this is the hardwood of Buddhism.

CONCLUSION.

Buddhism is decidedly not nihilism, and Nirvâna does not mean annihilation. Buddhism in its purest form is, more than any other religion, stated in philosophical terms, which, the more positively philosophical they are, will naturally appear to unphilosophical minds as mere negations.

Christians find it difficult to comprehend Buddhism, but the fact remains that what Christianity has been to Western peoples, Buddhism was to the nations of the East; and all the dissimilarities will in the end only serve to render the similarities that obtain between them the more remarkable.

While we are not blind to the great preferences of Christianity,

[1] See Karl Eugen Neumann, *Die Reden Gotamo Buddho's*, p. 304–325.

we must grant that Buddhism is a truly cosmopolitan religion. Buddhism can comprehend other religions and interpret their mythologies, but no mythology is wide enough to comprehend Buddhism. Buddhism is, as it were, religious mythology explained in scientific terms; it is the esoteric secret of all exoteric doctrines. It is the skeleton key which in its abstract simplicity fits all locks.

This is the reason why Buddhism can adapt itself to almost any condition and can satisfy the spiritual needs of great and small, high and low, of the learned as well as the uncultured. It offers food for thought to the philosopher, comfort to the afflicted, and affords a stay to those that struggle. It is a guide through the temptations of life and a lesson to those in danger of straying from the right path. And yet it demands no belief in the impossible; it dispenses with miracles, it assumes no authority except the illumination of a right comprehension of the facts of existence.

EDITOR.

LITERARY CORRESPONDENCE.

FRANCE.

IT IS unnecessary to repeat the criticism which I have already given of the latest remarkable work of M. Th. Ribot, *La Psychologie des Sentiments*. I desire merely to call attention to the approved merits of his method, to his steadfastness in adhering to one point of view and in supporting his conclusions upon a few dominating ideas, whose ramifications he unerringly follows, and finally to the decision which he evinces in his criticism of the numerous theories that come in his path, and between which he is obliged to choose.

To abide as rigidly as possible by the naked statement of facts, to strive constantly to single out the simple and primitive from the complex and secondary, such is the maxim followed by M. Ribot. Evolution supplies him with his instrument of analysis — the sound principle that all the luxuriant embroidery of higher life has been raised upon the canvas background of fundamental tendencies. And as these tendencies, which are the very basis of our being, are manifested in movement, the motor element can serve us in the construction of a theory of the great psychological facts.

Conformably to this conception M. Ribot does not hesitate to declare that the motor manifestations are the essential thing in the sphere of sentiment, that "what are called agreeable or painful states constitute but the superficial portion of affective life, the lowermost element of which reposes on tendencies, appetites, needs, desires, which are translated by movements." The doctrine thus clearly formulated serves him as a guiding thread in all his studies,

whether he is dealing with subjects of general psychology (pleasure and pain, emotion and affective memory) or whether he is engaged with subjects of special psychology, such as the instinct of preservation, sympathy, the sexual instinct, social and moral instincts or religious, æsthetical, and intellectual sentiments.

M. Ribot has given us a motor theory of *attention*. He will give us later perhaps a motor theory of *imagination*. We hope it will be permitted him to explore in this manner the whole domain of psychology. In any event, he will have left upon this department of inquiry a strong impress, will have cleared up many obscure problems, and generally advanced solutions even where it has not been his lot to discover them definitively.

* * *

But yesterday the miracle of the world was *life*, to-day it is *consciousness*. The physiologists, and with them Claude Bernard, had regarded life as an irreducible property; afterwards it was sought to reduce it to terms of physics and chemistry, and one is inclined to think that the problem has been approximately solved after having read the extremely valuable work of M. F. Le Dantec, *Théorie nouvelle de la vie*. M. Le Dantec progressively studies the life of monoplastidules, or elementary life, then that of polyplastidules, or life properly so called, and concludes with a few pages upon psychic life. I cannot enter into the details of this work. It will be sufficient to emphasise the clear and new views of the author on life and death, and to mention the two principal conclusions of his book: (1) that psychic life is an epiphenomenon of physiological life, all things going forward physiologically as if consciousness did not exist at all; and (2) that in everything affecting the senses of observing living beings there is nothing transcending the natural laws established for gross matter, that is to say, the laws of physics and chemistry. M. Le Dantec is free from all dogmatism. His work will no doubt be widely noticed by biologists and philosophers.

I should make good an omission which I have made of an interesting volume by M. A. Sabatier, *Essai sur l'immortalité au point*

de vue du naturalisme évolutioniste.[1] The difficulty of the spiritualist conceptions regarding the survival of the ego has, as we know, always been the "realising," or the infusing of palpable life into, the soul, which at the same time it is sought to make immaterial and virtual, and to keep one and indivisible. M. Sabatier has sought to transcend this obstacle by imagining an ultra-terrestrial plasma as the physical vehicle of immortality. This plasma would be at once matter and space, life and spirit; the nervous centres would play with respect to it the rôle of accumulators, or condensers, creating conscious personality, and this personality once created could be affixed after death to a new organism capable of maintaining its integrity and even of increasing its energy.

The hypothesis of an ultra-terrestrial plasma is interesting, but it is not easy to conceive what would become of the diffuse psychical states which are imagined apart from all conscious subjects, nor how consciousness, if it depends on the association of nervous elements, could survive their dissociation.

There has also been much talk of two works by M. A. De Rochas, *L'extérioritation de la sensibilité* and *L'extériorisation de la motricité.*[2] The experiments which are mentioned in these works should not be confounded with the "miracles" performed in the séances of the spiritists. M. de Rochas is a man of worth and an inquirer of sincerity. Nevertheless, he does not take sufficient precautions against suggestion and fraud. I have recently learned from well-informed persons, that his celebrated subject, his medium, had—after imbibing—revealed some of his methods. M. de Rochas himself has exposed some of these impositions, but it does not appear that he has discovered them all. His facts have not been sufficiently corroborated to permit his hazarding the rearing of a structure thereon. Does this mean that one must deny without hearing, and that no properties of nervous energy remain to be discovered? Not at all. We have simply to leave certain questions open, so as not to adopt precipitate and false solutions.

[1] Fishbacher, publisher.

[2] Chamuel, publisher. The remaining works are published by Felix Alcan.

A communication addressed to the Munich Congress by Dr. Baraduc marks the beginning of a new order of researches, simultaneously pursued in Paris by a young scientist, M. Radel, concerning whose work our journals published last August some brief mention. M. Baraduc flatters himself that he has photographed thought, and M. Radel that he has photographed dreams. That is to say, photography is said to have revealed to them the fact that there exist modes of exchanging nervous energy with the external environment, and also particular forms of the discharge of that energy. We are in the way here, should these doubtful facts be true, of not only giving precise material and form to intelligence, but of more proximately grasping the physiological fact corresponding to the psychological operation.

* * *

M. De Roberty gives us the first volume of his *Ethique, Le Bien et le Mal*,—an interesting work, as are all those of this author, both by its contents and its signification in the philosophical whole to which it belongs. It is less a systematically constructed book than a series of controversial articles, in which the enthusiasm of the writer breathes of the spirit of life, yet not without the sacrifice of lucidity. M. de Roberty predicates with many others the relativity of morals and pronounces future "immorality" as a benefaction. The term immorality signifies here, so far as I can understand, nothing but the end of special systems of morals, which are an obstacle to evolution, and not the end of all norms and of all authority. To every social organisation there corresponds, of necessity, an organisation of ideas and emotions which is morality itself. And it is thus that ethics is modified, but not without the establishment in the course of evolution of principles which thenceforth become, as I have elsewhere shown, the axis about which a new society grows up.

But we touch here the kernel of M. Roberty's book. He has proposed to answer mainly two questions: first, what is the place of morals with respect to biology, with respect to psychology, and with respect to sociology. Secondly, of the moral fact and social fact, which is prior?

To biology, he concludes, rudimentary psychism belongs; to sociology collective psychism, the study of which will take up the second volume of his *Ethique*. The florescence of life—religion, philosophy, science, and art—is not entirely due to the normal evolution of biological psychism; it results from the fusion of purely vital energies and of social forces derived from the biological order. Psychology, accordingly, would not be an abstract science (a science of *being*), taking rank after cerebral physiology; but it would be a concrete discipline (a science of *becoming*), a body of knowledge derived from the two conjoint sciences of biology and sociology.

As to the moral facts and the social facts it is to be said that the first engender the second rather than that they are derived from them. The social facts are the form in which the moral facts are clothed; in reality we have here the same order of phenomena, subject to a continuous evolution, in which, however, we must distinguish two aspects, the moral, which is within, and the social, which is without. I find nothing to object to in this conception. It has seemed to me clear for a long time that the same needs have given rise to society simultaneously with morality; that the development of both has proceeded upon the basis of our fundamental organic tendencies; and finally, that sociology is always broader than historical systems of morality, so that the latter constantly tend to conform to the former, and the psychological states of social individuals, that is to say, ends or duties, to agree more and more exactly with one another, instead of becoming antagonistic.

* * *

M. Louis Couturat, who gives us a book on *L'Infini mathématique*, is certainly a scholarly and distinguished author, yet one of a class in whom philosophical studies have strangely warped the geometrical spirit. His object has been to prove, as before him others have sought to do, that we can think and comprehend the infinite although it is not representable. He has sought this proof by a criticism of the data of mathematical analysis. But he commits in my judgment two fundamental errors. The first is the mistaking of the true nature of arithmetic. Arithmetic is a mere instrument of precision, the perfecting of which, as determined by

its application to complex questions and as pushed higher and higher by necessity, are recorded in history by the successive consideration of irrational numbers, imaginary numbers, limits, etc. It is not permissible to ascribe to the artifices which support it a mysterious value, or to attempt to objectify and hypostatise the purely logical conceptions which analysis has introduced into thought.

The second error is the attributing a special efficacy to magnitude, such that analysis is made to repose upon the idea of *magnitude* and not upon the idea of *number*. As if magnitude signified anything, so long as it remains undetermined, that is, unexpressed by means of numbers, which are the precise elements of its determination!

Cannot M. Couturat see that his induction ultimately leads to quite arbitrary changes in the true signification of words, and that the infinite which he has in mind and seeks to demonstrate is not at all the mathematical infinite?

* * *

I hasten on to the new publication of M. J. STRADA, *Jésus et l'ère de la science, la véritable histoire de Jésus*, and I take this occasion of calling the attention of my readers to the author himself, who is a philosopher by nature. M. Strada has had the singular fortune of passing his life almost entirely neglected by the official philosophical public, although he has published more than twenty books on philosophy, social science, and history, not to mention an enormous poetical work, *L'Epopée humaine*, which already embraces nineteen volumes. He undoubtedly owes the neglect which he has brought upon himself—if it can be called such—to the strange forms of his language and to the intricate style of his dialectics. I say that he has been neglected; I do not say that he has been overlooked or ignored. He has himself claimed priority for the theory of *idées-forces*, which M. Fouillée developed with such talent and originality.

Twenty years ago I read the *Ultimum Organum* of M. Strada, which had appeared ten years previously. I was struck with the work and spoke of it in my first modest and imperfect maiden effort.

Quite recently I reviewed in the *Revue Philosophique* another work by the same author, *La loi de l'histoire*, and I could not help remarking the agreement of Strada's law with that of Comte. M. Strada explains history, as did Comte, by intelligence; and the sequence of the methods of mind established by him—fideism, with faith as its criterion, rationalism with evidence, and impersonalism with facts—recalls the three ages, the theological, metaphysical, and positive, of his predecessor. M. Strada has made a novel and felicitous point where he reproaches positivism and modern science with having accepted experiment as a criterion, a thing which is for him merely a methodological instrument. The proper criterion, he contends, is found neither in experiment, in the syllogism, nor in mathematics, which are a simple means of reaching facts; it is found in the *Fact* alone. It is the *Necessary Fact* which is the criterion, not fluctuating and changing man, and hence the name of methodological impersonalism, or of the impersonal method, which he has given to his doctrine.

Yet the fact, one will say, is the very thing that experiment seeks to disclose. And the confusion is not as great as M. Strada imagines. It is true that in declaring experiment to be the criterion, we are in danger of excluding all metaphysics from knowledge, and the syllogism alone, according to M. Strada, reaches the metaphysical fact which he wishes to restore. There is a broad field for discussion regarding the scope ascribed by him to the syllogism and regarding his handling of the antinomies which leads him to affirm God to be the Pre-antinomic. But this discussion would be at present beyond our scope, and I return to the *Jésus* which I mentioned above. The work is large and interesting. Many people will be offended by it, and yet the restoration of the true history of Jesus which M. Strada attempts, taking his stand solely on the text of the Gospels, directly and respectfully consulted, appears to me an extremely probable one. Nothing is more striking than the sureness and frankness with which the author substitutes what he calls the fact-mediator for the deified mediators, such as Buddha, Jesus, and Mohammed, that is to say, a religion of science for fideisms of all sorts. M. Strada will finish this work in a new

volume, "The Religion of Science," which will certainly rank among the most interesting. An eloquent and convinced writer, with a zeal amounting almost to passion, he not only impresses and moves his public but also forces them to think. He knows philosophy as it is not known now-a-days, and handles language with an energy far above the ordinary. Although I do not give my full adhesion to his doctrine, I am ready to render him this homage and to bring his work to the attention of readers who doubtless are unaware of his existence.

LUCIEN ARRÉAT.

PARIS.

CRITICISMS AND DISCUSSIONS.

PANLOGISM.

A reply to Dr. Carus's reply (October number of *The Monist*) would involve a very lengthy paper. The issues raised open up a controversy of very extensive scope: in fact practically all the questions differently answered by the "monistic positivism" of my kindly and accomplished critic and by my own Neo-Leibnitzian monadism. And in view of the space occupied by me in the last *Monist*, I lack the effrontery to pen the long essay required.

Let me say here that all the issues are treated—some at considerable length—in my *Riddle of the Universe* (Arnold: London and New York). Those, however, who desire a succinct statement of the *ground-principles* of my system, may be referred to the essay in the July number of the *Free Review* (London: Swan, Sonnenschein) entitled "The New Monadism." Pending the publication of my developed system, I have nothing to add to the arguments there advanced. Let me observe in passing that no one who reads this essay will echo Dr. Carus's opinion that my monads are "substances which, for the sake of ridding them of gross materiality, have been reduced to atomic size." Size is a space-attribute, and space, in my view, only a form of perception *and ideation* of a monad. The monad or self (Kant's "transcendental subject") is not in space, but contrariwise space is in it. I have dealt in the *Riddle* with this issue at length and cannot understand how one who reads it can fail to follow my meaning.

I must just glance in passing at Dr. Carus's theory of Immortality. Jesus is immortal because his words are immortal. "The words of Jesus are his soul." "Christ lives where the word of Christ is received and where it becomes the motive of conduct." This is a *Comtean* immortality only. Jesus or John Smith is not destined to enjoy or suffer a perpetuity of *conscious* life: they only pass on ideas or "thought-forms." Let me point out some considerations bearing on this doctrine:

1. The immortality is verbal. Indeed, it is not even this. The slow freezing of the planet, nay, even the perishing of certain human stocks, would terminate it! Jesus would no longer persist, were there no terrestrial beings to repeat his words or act on his teachings.

2. It cannot be said that the "thought-forms" would even persist *as long as* men lived to receive them. And why? Because no two people can "think" exactly the same thoughts; *there are as many Christianities as there are persons*, and the name Christianity stands not for any definite persisting standpoint, but for a myriad-faced, *always changing process*. The "thought-forms" of Jesus, Dr. Carus, a bishop, and a tramp (all "Christians") are so many different psychological phenomena; and labelling certain vaguely-resembling portions of these phenomena as "Christianity," does not at all hide their vast differences. Jesus on Comtean lines does not persist at all—he has merely *started* an ethico-psychological process which is always changing its shape. Ideas ABOUT Jesus's ideas are not the ideas of Jesus!

3. The "immortality" is of no ethical value. Men who do not believe that *they* persist after physical death will not—taking communities into view—uselessly vex themselves with painful self-culture or "progress." They will degenerate; (a consideration, however, of no relevance where the *proofs* of persistence are being discussed).

There is much else to be said against this Comtean view of immortality, but the above considerations will for the present suffice. E. DOUGLAS FAWCETT.

EDITORIAL COMMENTS.

I should have preferred to publish Mr. Fawcett's comments on my reply to his "Panlogism" without further remarks, were it not that his explanations of my view of immortality might then seem to be acknowledged by me as being correctly stated. Therefore, I wish to add a few words which shall set the reader right, at least as to the main point of the subject.

Mr. Fawcett, who regards my view of immortality as Comtean, still insists on making a difference between a man's self and his soul-forms. He grants to some extent the immortality of the latter, but he thinks that the repudiation of a self-soul as a separate entity renders it illusory. Mr. Fawcett forgets to tell us what a soul would be without its peculiar ideas, sentiments, and aspirations. He assumes the existence of two things, (1) a soul in and for itself, a monad, and (2) the life and deeds of a man. Thus Jesus would be (1) a Jesus-monad and (2) his life consisting of his preachings and the moral example he set for mankind.

According to the immortality which I proclaim, Jesus is not a self in itself which preached certain theories, but his life, consisting of his preachings and his actions, was he himself. Jesus did not have the logos, but he was the logos, the logos being the truths which appeared in him; and this logos according to the Christian doctrines of the Fourth Gospel is an eternal, omnipresent reality in the constitution of the world. The logos was before Jesus was born and continued to exist after his death. It was at the beginning and will remain forever and aye even though this earth of ours break to pieces.

Mr. Fawcett would be clearer about the true nature of the self if he only proposed to himself the question, "What am I?" "What is Jesus?" "What is Mr. Smith?" He will find upon a proper analysis that every man consists of the memories of all deeds done in his own life as well as in the lives of his ancestors. He is the product of a long process of evolution, and as he is the continuation of the past, so in the future he will be the continuation of the present. Every organ-

ism is a system of memories, and memories are the immortalised previous reactions upon the outer world; they are the preserved deed-forms of innumerable acts committed in past ages; and there is no surd left which might give occasion to the belief in a soul-monad or a self-soul, a transcendental subject, a metaphysical substratum of our being, assumed to exist in addition to the real facts of our soul-life.

It is true that everything in the world, man's soul included, is subject to change, but it is a change in which every event is preserved forever, and the laws of nature are immutable and eternal. There is a change in identity and an identity in change; there is permanence in transiency, and transiency in impermanence. The belief in something that would be absolutely permanent and absolutely self-identical (whether we call it monad, or self, or subject, or âtman) is as gratuitous as the belief in absolute transiency and absolute change.

Experience teaches us that this world does not consist of matter and motion alone, but there is a formative factor which conditions the forms, the qualities, the suchness of things. The world is regulated by law, and its formative principle depends upon definite and intrinsically necessary relations which we develop in the so-called formal sciences, especially mathematics, arithmetic, and logic. The arrangement of thoughts cannot be made arbitrarily but must, in order to agree with the reality that surrounds us, follow definite lines, and in the same way every action determines its consequences with the same necessity that causes the circumference of a circle to remain everywhere at an equal distance from its centre. The totality of such conditions as constitute the cosmic order of the world is in its religious application called the logos, and the logos is an immaterial presence, and an inalienable feature of existence, the actuality of which does not depend upon the existence of supposititious monads, or subjects, or selfs or what not.

Mr. Fawcett calls that philosophy which upholds the omnipresent reality of the logos, panlogism, and tries to replace it by his monadology. He tries to make out that the ideas we think are foreign to our being and that for instance the Logos that became flesh in Jesus would be of no account unless Jesus's soul consisted of a monad which would have to be regarded as his self. In recapitulating my views of immortality Mr. Fawcett tacitly assumes that I believe in the annihilation of this monad self in death, while I claim that such a monad self has no existence and can therefore not be annihilated, while the real facts of which we consist remain living and effective presences in our after-life.

And our after-life is as little unconscious as our present life which is the continuation of our former lives. To be sure there is a break in the continuity of consciousness in death; but this break is on the same principle as the break that occurs in sleep. Every morning we wake with fresh consciousness and renewed energy, yet the memories of our former life-experiences remain the same and their preservation constitutes the preservation of our being. Thus every new life starts with a fresh consciousness, but if we analyse its organisation we find that it consists of innumerable memories of deeds done since the remotest past in its ancestral existence.

As to the indestructibility of everything that is valuable, true, and good, we trust that if this world breaks to pieces, the Logos will reappear in other worlds. Nay, we believe that on other planets on which the same conditions prevail as on this, our earth, the Logos is present now, and it makes little difference whether he be Joshua, of the tribe of Juda, or Gotama, of the tribe of Shakya.

Whether or not Mr. Fawcett has overcome panlogism I must leave our readers to judge for themselves.

P. C.

THE INTERNATIONAL SCIENTIFIC CATALOGUE, AND THE DECIMAL SYSTEM OF CLASSIFICATION.

The most notable bibliographical event of the year was the holding of an International Conference at London in July last for the purpose of considering the preparation and publication of a complete international catalogue of scientific literature. The germ of this idea originated forty years ago with a proposition made at the British Association by a distinguished American scientist, Professor Henry, and was partly realised by the "Catalogue of Scientific Papers" issued since 1867 by the Royal Society and designed to embrace all the purely scientific literature published since 1800, arranged according to authors' names. The proportions which this catalogue soon assumed, and its unavoidable bibliographical deficiencies, subsequently determined the Royal Society to undertake the preparation of a *complete systematic catalogue of all the world's publications in pure science*, arranged not only according to authors but also according to subject-matter. It was quickly seen, however, that the enormous magnitude of the plan far transcended the powers of a single organisation, and accordingly a circular was issued to all the great learned bodies and civilised governments of the world, inviting their co-operation in the consideration and execution of the scheme. For the details of the Conference we must refer our readers to the excellent report by Carl Junker in the *Centralblatt für Bibliothekswesen*, Leipsic, Vol. XIII., page 505, to the London *Academy* for August 1st, to the *Library Journal*, New York, for August and November, 1896, and also to the contemporaneous files of *Science* and *Nature*. We have only to remark that it was decided the Catalogue should be restricted to "pure" science, that its official language should be English, that it should be issued both by cards and periodical volumes, and that all the difficult questions involved in its preparation should be left to a special international committee, in charge of a central bureau at London.

The most important of all these questions, and the one that provoked the most discussion, concerned the system of classification to be adopted. The decimal system of Melvil Dewey, now director of the New York State Library, was suggested with modifications. Dewey's system, which has been in practical use for over twenty years, is so well known, so widely adopted, and recently been the subject of so much controversy, that little explanation of it is necessary. It is in use in many of the middle-sized and in most of the smaller libraries of this country, and was recently (1895) enthusiastically adopted by the International Institute of Bibliography at Brussels, Belgium, which has now its permanent working bureau. For a simple and brief account we could recommend no better source of information than Publication No. 5 of the Belgium Office (Hotel Ravenstein, Brussels), seeing that Dewey's own book is rather bulky and too detailed for the general reader.

The principle of the system is that of dividing all knowledge into ten main *bibliographical* branches denoted by the numerical characters from 0 to 9, of subdividing these again into ten more and so on *ad infinitum*, so that each branch of knowledge and each mode of knowledge has its definite and unvarying characteristic (just as a logarithm has), which can be interpreted at once by its place in the ordinal series and by the help of a comparatively simple general index. Thus 0 denotes General Works, 1 Philosophy, 2 Religion, 3 Sociology, 4 Philology, 5 Pure Science, 6 Applied Science, etc. By another subdivision, say of 5, we have respectively 0, 1, 2, 3, 4, etc. for General Science, Mathematics, Astronomy, Physics, Chemistry, etc., and by another subdivision, 530 means General Physics, 531 Mechanics, 532 Hydrostatics, etc., 535 Optics. There are further a few special marks for geographical and historical subdivisions, consisting of parentheses, colons, etc., so that 535.09(44.04), for example, is easily read as "the history of optics in France during the Revolution." The whole practical mechanism of the system, which admits of specialisation by subdivision in the enormous ratio of the powers of 10, is simply an alphabetical index and tables of general and special headings, which are repetitionary in principle. Its power and uses are not restricted to bibliography, but may be advantageously extended to *Indices Rerum*, etc.

It is evident that the system *apparently* involves a *classification of the sciences*, and this seems to have been the main ground of objection to the scheme at the London Conference, which curiously enough came principally from librarians, who have least to bother with questions of philosophy. It should not, however, be viewed as such, but should be regarded merely as what it is, *a practical scheme for arranging and indexing books*. Consequently, it can never, as has been claimed, hamper the advancement of science; for however false and illogical Dewey's classification of knowledge may be, the arrangement of books in a catalogue or on a library's shelves can at most only give difficulties to the arranging librarian or to the seeker—it can in no essential manner affect the progress of science. A perfect classification of the sciences we shall never have, and there is infinitely less probability that we shall ever have a perfect bibliographical system, for knowledge is so interrelated, its gradations and shadings are so subtle, and the caprices of authors are so great, that it is safe to say bibliographers will always be presented with substantially the same difficulties as they are to-day. The sole question is that of practical flexibility, ease and precision of consultation. These qualities the Dewey system seems to combine in a more eminent degree than any existing system, and in view of the momentous significance and inestimable practical value of the proposed International Catalogue, it is well that its merits should be strongly insisted upon and its defects thoroughly examined before rejection or adoption. At any rate we should bear in mind here that we are not concerned with a rigid philosophical scheme for classifying the sciences, but with a practical system of bibliography having no bearing whatever on the development of research.

In the first place, then, although we should not claim for the Decimal Classifi-

cation the merits of an absolute Real Character, yet there is no denying that it is essentially ideological in structure, and hence international. Secondly, it furnishes not only a bibliographical nomenclature but also a bibliographical notation which can be mechanically handled. Lastly, its power of expansion and meeting the growing needs of specialisation is unlimited, while the resultant ramifications of the system are symmetrical and entail little additional mnemonic burdens.

As to the defects they seem to pertain largely to matters of library economy, as the spatial separation of subjects nearly related (e. g., Philology and Literature), the decision of the proper category to which a book belongs, say Money or Finance, Applied Electricity and Mechanical Engineering, (a very elusive matter, generally inherent in the book and not in the system,) the treatment of subjects wherein the alphabetical system seems intrinsically indispensable, as Biography, etc. For the recital of these defects we may refer the reader to an impartial paper by W. L. R. Gifford in the *Library Journal* for November, 1896, to a letter by A. G. S. Josephson in *Science*, September 4, 1896, and to an article published last summer by M. L. Polain in the *Revue des Bibliothèques*. In its favor may be read the laudatory articles of C. Richert in the *Revue Scientifique* for April 11th and July 11th, 1896, the paper of W. E. Hoyle in *Natural Science* for July, 1896, and that of Marcel Baudouin in the *Revue Scientifique* for May 30, 1896, as also to the publications generally of the Belgian International Office.

As might be expected, the opponents of Dewey's system are strongest in the United States. And the opposition is mainly from the librarians of our large libraries, who have greater difficulties to compose and in many instances have systems of their own. Although claimed to be in use in one thousand libraries in the United States, it is said these libraries are small and not of high standing. The opposition of the great librarians should certainly be weighed by the Catalogue Committee, *in all its phases*. Furthermore, we have the authority of the above-mentioned writer in *Science* that in Amherst College and Columbia University where the system was first used, "it has all been made over again."

Be that as it may, the Decimal System certainly contains the germ of a universal bibliographical notation and it is extremely probable that in one form or another it will be adopted for the new International Catalogue. Being restricted to one main division, that of Pure Science, it will avoid some of the difficulties that have perplexed librarians but bring additional others in its train. In itself the question is of considerable importance, reaching far beyond that of mere bibliographical interest, and deserves the serious consideration of all scientific workers.

THOMAS J. MCCORMACK.

BOOK REVIEWS.

THE PRIMARY FACTORS OF ORGANIC EVOLUTION. By *E. D. Cope, Ph. D.* Chicago: The Open Court Publishing Co. 1896. Pages, 547. Cuts, 121. Price $2.00.

In the year 1866 there appeared in the *Transactions of the American Philosophical Society* a paper by Professor Cope on the Cyprinoid Fishes. Among the conclusions at which he arrived in this paper were the following: The relation between the generalisation or specialisation of a type and its future progress; the parallelism between ontogenesis and phylogenesis, though he did not use these terms; the results of acceleration and retardation; etc.

The character of the results of this investigation are quoted to show the tendency of thought of the writer. While others were discussing the truth or falsehood of the theory of evolution, or its applicability in special cases, he had turned his attention to the laws of working of the process.

In 1871 in a paper on "The Method of Creation of Organic Types" he propounded a theory of "growth-force" or "bathmism"; and showed how this force, located by effort or use at certain points of the body, produces progressive evolution. Conversely disuse results in degeneration. In this essay we find also the germ, at least, of his later views concerning the importance of consciousness in evolution as the means of directing or locating this use or effort.

These are only the beginnings of a long series of papers which Professor Cope has contributed to the *American Naturalist* and other periodicals during the last thirty years. They are all characterised by the same effort to discover not merely the mode but also the causes of evolution, or more especially of variation.

In 1887 he republished the results of many of these articles in his *Origin of the Fittest*. In this book he strongly opposes the "omnipotence" of natural selection. "Selection," he says, "requires alternatives, and these are the products of "variation. Great obscurity has arisen from the supposition that natural selection "can *originate* anything, and the obscurity has not been lessened by the assertion "often made that these variations are due to inheritance. What is inheritance but "repetition of characters possessed by some (no matter what) ancestor; and if so,

"where did that ancestor obtain the peculiarity? The origin of variation is thus "only thrown upon an earlier period."

The present book would seem to have been called forth by the prominence of Weismann's theory, which is diametrically opposed to many of the author's views. It is a discussion of the theories of Preformation and of Epigenesis. "In one of these," he says, "the variations of organisms which constitute progressive and re- "trogressive evolution appear fortuitously, and those which are beneficial survive "by natural selection, while those which are not so, disappear. Characters both "beneficial and useless or harmless, which are acquired by the adult organism, are "transmitted to the young, so that no education in habit or structure acquired by "the adult has any influence in altering the course of evolution. This is the doc- "trine of Preformation. From this point of view the cause of the variations of "organisms has yet to be discovered.

"The other point of view sees in variation the direct result of stimuli from "within or without the organism; and holds that evolution consists of the inheri- "tance of such variations and the survival of the fit through natural selection. "This is the doctrine of Epigenesis. To this I would add that in so far as sensa- "tions or states of consciousness are present, they constitute a factor in the pro- "cess, since they enable an organism to modify or change its stimuli. . . . My aim "will be to show in the first place, that variations of character are the results of "physical causes; and second, that such variations are inherited." (Pp. 13, 14.)

The first chapter of the book treats of variation. Here a large number of cases are adduced to show that "variations are not promiscuous or multifarious, but are of certain definite kinds or in certain directions. (P. 22.) The second chapter containing a little less than one hundred pages, is devoted to phylogeny; twenty-five of these to the more immediate ancestry of man.

This is one of the most interesting, and the most tantalising, of all the chapters of the book. The phylogeny of the classes of vertebrates is discussed in about sixty pages. The phylogenetic charts of the different classes are clear and not confused by unnecessary details. The author is a master of the science of recognising what he can afford to leave out; a science of which most writers seem to be woefully ignorant.

It is interesting to compare this chapter on phylogenesis with Haeckel's *Phylogenie der Wirbelthiere* published a year or more ago. The German scientist gives us a volume of some six hundred large pages. There is no attempt at condensation. The style is delightfully easy and flowing. Paper and ink are abundant and he writes for readers who do not like to be hurried. Wherever the actual ancestor of a line of descent has not yet been discovered, the author reconstructs a hypothetical ancestor. And this hypothetical ancestor "must have existed"; there is no doubt about his existence or characteristics. The connexion and the progressive modifications of the different lines are always clear.

Every new term is carefully explained. The gaps are bridged; the difficult

places smoothed and straightened; and we read easily, pleasantly, and without effort. The boundary line between the actual and the hypothetical, between the "is" and the "must be," is not always sharp. But we comfort ourselves with the thought that Professor Haeckel's guesses are very shrewd, and that they have usually been verified by later discoveries.

Our American writer, in his lines of vertebrate phylogenesis, sticks as closely as possible to the facts of palaeontology. The statements are very brief, the anatomical terms are rarely or never explained. New names for great groups of animals are introduced with the briefest definitions possible. The chapter is, it must be confessed, hard reading. And even when we have finished it we are not quite sure as to just what sort of animals, for example, the Cotylosauria were, or how they differed from the Theriodonta. And much the same is true, though in less degree, of the section on the phylogeny of mammals.

The account of the phylogeny of the horse closes with the statements that its "history may be duplicated in manner and mode, by the lines of the camels, the "dogs and bears, the cats, the beaver, etc." And "examination of all these lines "reveals a certain definiteness of end and directness of approach. We discover no "accession of characters which are afterward lost, as would naturally occur as a "result of undirected variation." (P. 149.)

One cannot but feel that the argument of the book would have been strengthened if the author had given us a history of others of these narrower lines of mammalian development, even at the expense of leaving out the discussion of vertebrates in general.

For in this chapter the author seems to the ordinary mind to have undertaken the impossible. The chapter should be expanded to a volume, and "writ large" so that all could understand. When we remember how largely the material for mammalian phylogeny has been discovered in America and studied by American palaeontologists; when we notice the striking similarity between many of the charts of Haeckel's *Phylogenie* and those published more than ten years ago in Cope's *Origin of the Fittest;* when we remember further the numerous references to the author's investigations by Professor Zittel in his *Palaeontology;* we feel that we have a just claim on the author of this book for a work on Vertebrate Phylogeny, and that "t'were well 'twere done quickly."

The third chapter discusses the parallelism between ontogeny and phylogeny, and the fourth chapter treats of Katagenesis or degeneration. Here the relation between degeneration and akinetogenesis, or lack of use or effort, is well presented.

In the second part of the work the author treats of the Causes of Variation. He says "I propose to cite examples of the direct modifying effect of external in-"fluences on the characters of individual animals and plants. These influences "fall naturally into two classes, viz., the physico-chemical (molecular), and the "mechanical (molar). The modifications so presented are supposed to be the re-"sult of the action of the causes in question continued throughout geologic time.

"To the two types of influence which thus express themselves in evolution, I have "given the names Physiogenesis and Kinetogenesis."

The chapter on physiogenesis contains a very interesting series of observations of the effect of light and color on animal coloration. But the chapter might well have been longer. The author has selected rather too sparingly from the wealth of illustrations which he had at his command.

Chapter VI. treats of Kinetogenesis or the effects of use and disuse. This is the longest and, all in all, the most interesting chapter in the book. The author discusses the shells of mollusks, the effects of use on muscles, the results of impacts and strains on bones, the origin of dental structures, and other important subjects. The argument is clear, strong, and convincing. The chapter contains also an admirable account of the origin of osseous vertebrae.

The third part of the volume treats of the inheritance of variation. In the chapter on Heredity the hypothesis concerning the mode of inheritance of acquired characteristics is stated with remarkable clearness and precision. The author says: "The effects of use and disuse are two-fold, viz.: the effect on the soma, and the "effect on the germ-plasma. Those who sustain the view that acquired character-"istics are inherited, must, I believe, understand it as thus stated. The character "must be potentially acquired by the germ-plasma as well as actually by the soma. "Those who insist that acquired characters are not inherited forget that the char-"acter acquired by the soma is identical with that acquired by the germ-plasma, "so that the character acquired by the former is inherited, but not directly. It is "acquired contemporaneously by the germ-plasma, and inherited from it. There "is then truth in the two apparently opposed positions, and they appear to me to "be harmonised by the doctrine above laid down, which I have called the Theory "of Diplogenesis, in allusion to the double destination of the effects of use and dis-"use in inheritance." (P. 443.)

The whole chapter is so full of facts, thought, and suggestion that no one quotation can do it justice. But some of the evidence adduced for the inheritance of acquired characters seems decidedly weak.

The chapter on the Energy of Evolution is especially interesting for its logical tendency. The author divides the energies manifested by living beings into those which "tend away from, and those which tend toward, the phenomena of life." The latter or anagenetic class is exclusively vital, and tends to upward progress, in the organic sense, that is toward the increasing control of its environment by the organism. The former class, composed of the catagenetic energies, is physical and chemical. "The catagenetic energies tend to the creation of a stable equilibrium "of matter, in which molar motion is not produced from within, and sensation is "impossible. In popular language, the one class of energies tends to life; the "other to death." (P. 475.) In another passage he says, "I have given to that "energy which is displayed by the plant in the elaboration of living from non-living "matter the name of antichemism."

The discussion of this apparent "dualism" is exceedingly suggestive, but it is too long to give in full, and mere quotations would only do it injustice.

One great merit of the author's writings is that, while he never undervalues the importance of mechanical processes in evolution, he always emphasises the importance of mind. The Function of Consciousness is well treated in the tenth chapter. "Consciousness was coincident with the dawn of life." "It has pre-"ceded in time and in history the evolution of the greater part of plants and ani-"mals, both unicellular and multicellular. It appears also that, if kinetogenesis "be true, consciousness has been essential to a rising scale of organic evolution." "I think it possible to show that the true definition of life is, *energy directed by "sensibility, or by a mechanism which has originated under the direction of sensi-"bility.*" (Pp. 508–513.)

Every action was primitively the result of conscious effort and "the mechanism which does the work has developed as the result of the animal's exertions under stimuli."

This is good common sense and sound logic.

We must make just one more quotation: "Why should evolution be progres-"sive in the face of universal catagenesis? No other ground seems discoverable "but the presence of sensation or consciousness, which is, metaphysically speaking, "the protoplasm of mind. The two sensations of hunger and sex have furnished "the stimuli to internal and external activity, and memory, or experience with "natural selection, have been the guides. Mind and body have thus developed "contemporaneously and have mutually reacted. Without the co-operation of all "these factors, anagenesis seems impossible."

The book closes with a brief chapter on the Opinions of Neo-Lamarckians.

It is impossible to give in a brief space an adequate outline of such a book, for the wealth of facts and arguments, of new thoughts and suggestions, has to remain almost unnoticed. Professor Cope is a peculiarly suggestive writer. Old theories are viewed in a new light, are analysed or put in a new or modified form. Scattered all through the book are facts or hypotheses, sometimes bearing only very indirectly on the argument of the chapter, which are full of food for thought.

The chapter on Phylogeny ends with a section on the Law of the Unspecialised, a condensed but very clear presentation of the fact that higher types have always sprung from generalised forms. This law ought to be more widely promulgated in these days of extreme specialisation; for almost every one considers it a "dead letter." But, if true of physical evolution, and our author certainly very nearly demonstrates it, why should it be false in the evolution of the individual mind?

The frequency and importance of small size in progressive lines, especially in mammals, is stated in a single sentence of the same section. Other illustrations are the hypotheses concerning the origin of lungs and of bony vertebrae. The

hypothesis concerning the origin of the rhachitomous vertebra, and its illustration are especially worthy of notice in this connexion.

On page four hundred and forty-four sexual reproduction is considered advantageous "on account of its increased opportunity of variation." But is it yet certain that sexual reproduction does not work rather to hinder than to increase variation in the group or species? Does not the intercrossing of forms which have been exposed to different conditions, and which are therefore tending to diverge, result in holding the species as a whole somewhere near a golden mean of structure and progress? Is not one great danger of the intercrossing of closely related forms to be found in the fact that thus individual tendencies of variation are enhanced until they are abnormal and injurious? Indeed, does not nature, so to speak, have to keep a brake on the too rapid variation of the central, ascending phylogenetic line leading toward man, lest too many of its members rapidly become specialised and thus unproductive of anything higher? There are doubtless other, perhaps even more effective, hindrances to specialisation; but does not sexual reproduction work with these rather than against them? This, if we remember rightly, is Hatschek's view; but we may do him injustice.

The whole book is a marvel of condensation. It is a storehouse of facts, arguments, and suggestions; but it is so condensed that it is not easy reading. It is very pleasant to come across a scientific article or book where the chaff has been carefully removed. They are exceedingly rare, but even the virtue of condensation may be exaggerated into a fault. One must read it chapter by chapter. Like some condensed foods, it requires good powers of digestion in the user.

It is moreover the work of a thorough palaeontologist as well as profound student of comparative anatomy. Hence it has a special place and value.

But Weismann is a very skilful and wary antagonist, and seems to have a defence for every attack. If the palaeontologist can show that variation is really linear, Weismann's theory is disproved. And Professor Cope has given us lines of mammalian evolution, for example, traced out with great clearness; and they are proven with a vast amount of acumen, skill, and patient observation. It is dangerous to try to imagine just what a follower of Weismann would say about any subject for the theory is not only complex but also protean; it changes front to every new attack. But might he not make the following objection with some justice? Of all the formerly existing individuals of a series of species of vertebrates the palaeontologist has but few specimens and these collected from a comparatively narrow area. His material is too scanty to give him any adequate conception of the amount of variation of which the species at any one stage was capable. When more material has been collected the variation of any species at each stage may yet prove to be fortuitous. The forms which survive through successive periods of time, or through a series of stages of evolution naturally form more or less straight lines of variation, for these lines are favored by natural selection. But this linear survival is entirely compatible with a fortuitous variation. Is it not at

present an almost necessary result of the character of the two fields of study that the zoölogist should have the advantage in the study of variation, and the palaeontologist in the study of survival and hence of phylogeny? And that parallel lines of survival should be fostered in different groups by natural selection from fortuitous variation need not surprise us.

This objection may not apply to inferences drawn from the teeth of mammals for here the palaeontologist may have vastly more material than we have supposed. And the most devoted follower of Weismann must feel surprise that the lines of survival are so straight and with so few branches. We cannot fail to notice how largely the palaeontologists are Neo-Lamarckians.

In the emphasis placed on consciousness, will, and effort, this volume is a most valuable and timely contribution. Students of evolution have too generally represented not only vital processes but even life itself as almost or quite purely mechanical, molecular, or chemical. Hence they have either neglected or slurred all its mental aspects. They have sought the living among the dead until they forget what life is and what are its chief characteristics. For supremacy of mind over material, and finally over itself, is the evident goal of evolution. All such purely mechanical or chemical theories, when applied to human progress, necessarily proved misleading or useless.

But when life is defined as "energy directed by sensibility," each of its aspects has received its due emphasis. Well may the author claim that "from this point of view the study of the evolution of mind and its relation to the organic world assumes a new importance."

Now and then the book reminds us of the writings of the Apostle Paul; "in which are some things hard to be understood, which they that are unlearned wrest unto their own destruction." Bathmism we can remember, and its meaning also. But what of Statogenesis, Emphytogenesis, Autobathmogeny, Mnemogenesis, and Cryptopnoÿ? If even Mephistopheles had seen these and sundry other compounds which occur in the volume, he could hardly have found it in his heart to urge an unsuspecting student to "learn words."

Any one who will read this book carefully and thoughtfully cannot fail to have a new, and clearer, and more just, conception of the factors and the process of evolution; and will find his mind continually stimulated to think along new lines.

JOHN M. TYLER.

OSTWALD'S KLASSIKER DER EXAKTEN WISSENSCHAFTEN. A Serial Publication, at Present Embracing More than Eighty Works in Mathematics, Physics, Astronomy, Chemistry, Crystallography, Botany, and Physiology. Edited by Prof. Dr. *Arthur von Oettingen*. Leipsic: Wilhelm Engelmann.

The impression is a widespread one in the popular mind that novelty in science, like novelty in the practical arts, constitutes by the very fact and virtue of its novelty an advance upon the old, supplanting and undoing it. The popular mind,

and with it its reflex popular pedagogy, is in error here, in error principally by its inability to grasp the salient and fundamental features differentiating knowledge, and secondarily by its utter lack of sense for the exigencies of historical and cosmical development—a joint, or rather disjoint, mental condition which leads people lower in the scale of intelligence (say our school-boards) to welcome revisions of the multiplication-table with the same unfeigned delight that the biologist does modifications of Dr. Weismann's theory of heredity. The sciences exhibit varied degrees of *a priority* and formal rigor ranging from arithmetic to psychology, their development has not been contemporaneous, and consequently they are not all at the same stage of perfection. In some we can hope for but little more than new and ingenious presentations, while in others we may expect at any day astounding revelations. We must distinguish between the two classes of knowledge. In the former it is not likely that the same pitch of excellence will again be attained, that we shall ever again in these departments reach the same naturalness and power of thought or the same beauty of exposition—for the sufficient reason that genius will never again apply itself to these departments with equal fervor. The very necessity of such application is wanting, for a truth once discovered remains a truth forever, and is not in need of rediscovery. Such is one of the considerations which in certain branches of knowledge, and under certain restrictions, turns our glance to the past.

But there is another. In this decadent age, with its tendency to intellectual democracy, when every Tom, Dick, and Harry may yield to the unholy impulse to mutilate science, the prime necessity in the spirit which shapes research is a sane conservatism. Not a conservatism which cleaves slavishly to old ideals and methods, which apotheosises old models and stifles the impulses of originality, but a conservatism which ever keeps before the student's mind the marks of high achievement and lofty standards, and holds to his ears the memory-ring of true genius. We are concerned here merely with the plea, which all history confirms, that it is not given to every man and age to reach Olympian heights in their performances, but that some are preferred before others. That aggregation of the cosmic elements which went to make a Michael Angelo, a Kepler, a Shakespeare, or a Kant, is not compacted by the Divine Artificer or *Zeitgeist* in every age of the world's history, though it may be in the making to-day or to-morrow. Inevitably, therefore, and as it were by the very eccentricities of the universe, by the very conditions of intellectual evolution, we are led back to the Golden Ages of Science, Art, or Literature whenever we would seek our highest inspiration and culture.

Some such objects as these, at least on the æsthetical and theoretic side, it is the purpose of Ostwald's Series of Scientific Classics to promote. The series itself is, in its department, one of the most important and deserving enterprises which have been undertaken in recent years. It derives its name from its original editor, Dr. Ostwald, who, on the assumption of that post by Dr. Arthur von Oettingen, likewise an indefatigable scholar, has not ceased his collaboration, but still con-

tinues to enrich the series by selections, translations, and special editorial work. Having originated with a man who, as his recent utterances show, is keenly alive to the stupendous practical import of science, the philosophical, æsthetical, and purely historical ends which the series may primarily seem designed to satisfy, are extended in their significance so as to embrace broad practical aspects of the scientist's culture.

We shall now address ourselves to the contents of the series, beginning with mathematics, and taking up first the Calculus of Variations. It will be profitable here to quote, on the advantages of historical scientific study, the words of Robert Woodhouse, a Cambridge mathematician, the original pioneer in this department, who, in his *Treatise on Isoperimetrical Problems*, published in 1810, after mentioning the stimulus afforded to the student's curiosity and attention by a combination of historical and systematic researches, says:

"But other advantages, besides that of an excited attention, may accrue to "the student from the present plan. He will have an opportunity of observing "how a calculus, from simple beginnings, by easy steps, and seemingly the slight- "est improvements, is advanced to perfection; his curiosity, too, may be stimu- "lated to an examination of the works of the contemporaries of Newton; works "once read and celebrated: yet the writings of the Bernoullis are not antiquated "from loss of beauty, nor deserve neglect, either from obscurity, or clumsiness of "calculation, or shallowness of research. Their processes, indeed, are occasion- "ally somewhat long, and want the trim form of modern solution. They are not, "however, therefore the less adapted to the student, *who is solicitous for just and "full views of science*, rather than for neat novelties and mere store of results. In- "deed, *the authors who write near the beginnings of science are, in general, the most "instructive; they take the reader more along with them, show him the real difficul- "ties, and, which is the main point, teach him the subject, the way by which they "themselves learned it.*"[1]

For this study, and precisely on the subject Woodhouse had in mind, we have in numbers 46 and 47 of Ostwald's Series abundant material. The initial isoperimetrical problems of the Bernoullis are given, the *Methodus inveniendi* of Euler, the two papers of Lagrange, and the two of Legendre and Jacobi. Wholly apart from its scientific importance, there is scarcely a chapter in the history of research that can compare with that of the Calculus for Variations in its intensely human interest. The challenges and strife of the Bernoullis, ending in a bitter feud between the two brothers, the magnanimous generosity of Euler, at that time prince of European mathematicians, who withheld the publication of certain researches till the young Lagrange should publish his, that the latter might not be robbed of "one iota of the rightful fame" due to him for his exquisite solution—all combine to make this period of mathematical history entrancingly interesting. Euler's letter

[1] Italics are ours.

is a model of the scientific attitude. "Your analytical solution of the isoperimetrical problem," he writes to the boy who was thenceforth to share his laurels, "leaves "nothing to be desired in this department of inquiry, and I am delighted beyond "measure that it has been your lot to carry to the highest pitch of perfection a "theory which I have been almost the only one to cultivate from its inception."

Or take another incident. Of the numerous problems which John Bernoulli showered upon the mathematical world in the latter part of the seventeenth century, and which were generally supposed to have been aimed at his brother James, the most famous and the one fraught with the greatest significance for science, was that of the brachistocrone, or the curve of quickest descent. It was answered by Leibnitz, Newton, De l'Hospital, and by James Bernoulli, the latter of whom retorted by a counter-challenge involving a more general problem, and ended by adding that since it was unjust that any one should go unrecompensed for labor on behalf of another and to the detriment of his own affairs, a gentlemen for whom James would vouch pledged himself to give his brother meet praise and fifty ducats besides, provided the latter would furnish a solution of the problem within three months and publish the same within a year. The time-limit John did not take advantage of, but published his solution immediately, saying that "instead of three months it had only taken him three minutes to penetrate the whole mystery." But in one point he had erred. James, to the terror of his brother, increased his wagers in geometrical proportion, and when John ultimately refused to revise his solution on the plea that his time was much better occupied in making new discoveries, gave the crowning retort-courteous in the reply "that if in *three minutes* he had solved the whole mystery, surely *six minutes* more would not much diminish the number of his discoveries." The wrangling of the two brothers continued till the death of James. It had its dark sides, but from its very passion was unusually fruitful for science. We may add that the view of John's unfairness taken by English historians is not wholly accepted by Cantor. Not all the material of the Bernoulli feud is given in the two *Classics* under consideration, but only the initial programmata: the rest is devoted to the modern developments mentioned.

We have also to mention in mathematics the two *Treatises on Spherical Trigonometry* (No. 73) by Euler, which are fundamental in their department. The didactic works of Euler are available to-day as text-books; and, notably in trigonometry, little has been added to the science since his time. His diffuseness is scarce a fault, and it is a significant comment on the methods of discovery that, though many of his demonstrations lack the boasted modern rigor, yet the theorems themselves have generally withstood all assault. On the other hand, the naturalness and lucidity of his explanations might well be readopted in modern instruction. Euler traversed like a conqueror the entire domain of mathematics, transforming and augmenting it at every step. Creator and systematiser, he left everywhere his giant impress. With him, therefore, and particularly in our days of specialisation, intercourse is quickening and chastening.

The other mathematicians represented in the Series are Gauss, Steiner, Jacobi, Abel, Bravais, Laplace, Dirichlet, Charles Ivory, Rosenhain, and Göpel.

We come now to Physics. The first works to claim our attention are: (1) the *Dialogues* of Galileo, in three small volumes (Nos. 11, 24, and 25),[1] admirably translated by Dr. Oettingen, and (2) Huygens's *Treatise on Light*, translated by E. Lommel. The *Dialogues* of Galileo rank as one of the loftiest achievements of the human intellect. They are as perfect in their literary form as they are momentous in their contents, and mark the real beginning of modern science. "They did not," says Lagrange,[2] "procure for Galileo, during his lifetime, the celebrity of his discoveries in the heavens, but to-day they constitute the solidest and realest portion of his transcendent glory. The discovery of the satellites of Jupiter, of the phases of Venus, of the spots of the sun, etc., required but telescopes and assiduity; but extraordinary genius was necessary to disentangle the laws of nature from phenomena which philosophers had always had before their eyes, but whose explanation constantly eluded their efforts." We can grasp Galileo's gigantic performance only by transplanting ourselves to the time in which he lived, by contemplating its absolute intellectual dependence on authority, and by recollecting that he worked almost entirely without instruments. One is struck by his unfailing common sense and insistence on practical points of view, his grace and lucidity of presentation, his simplicity and directness (a point in which he is the direct opposite of Kepler), and by his skilful manipulation of the cumbersome mathematical methods of his time. The inspiration to be derived from these volumes is surpassed only by the insight which they afford into the workings of the archetypal inquiring mind. In this their psychological value they stand without a peer.

Huygens is the second brightest star in the scientific firmament of the sixteenth century. He continued and supplemented with equal genius the work of Galileo, and founded in his *Horologium Oscillatorium* the second parallel development of mechanical ideas which ended in the modern doctrine of energy. He is represented in Ostwald's Series by his famous *Traité de la Lumiere*, which laid the founeations of the modern undulatory theory of light and which shows at their best the brilliant qualities of his mind. So powerful was the thrall of Newton's genius—even on its mightiest side it deadened the mathematical development of England during a whole century—that under the shadow of the corpuscular theory Huygens's ideas, despite their simplicity, remained undeveloped for fully three generations. The historical significance of the *Treatise* goes without saying; its disciplinary value is equally high. The masterly exposition of the facts and law of double refraction in Chapter V., says Lommel, is instructionally superior to that of the best of modern text-books. The *Horologium Oscillatorium* is missing from the series, but it is hoped the deficiency will soon be supplied.

[1] The prices of the volumes of the series vary according to the size. Full catalogues may be obtained by addressing W. Engelmann, Verlagsbuchhandlung, Leipsic, Germany.

[2] *Mécanique Analytique*, Vol. I., p. 237, Collected Works, Paris, 1888.

Notable, also, are the *New Magdeburg Experiments* of Otto von Guericke (No. 59), with their quaint drawings, their ponderous and costly equipments (the Bürgermeister spent 20,000 thalers on his apparatus and received as honorarium for his published work only a few free copies), and lastly with their delightful glimpses into the industrial life of the seventeenth century. The third book only of the work is published and contains the experiments on atmospheric pressure substantially as they are given to-day in the elementary school-books.

In No. 57 we have Fahrenheit, Réaumur, and Celsius's papers on *Thermometry*. It is curious to note that the mark 100° was originally placed by Celsius at the freezing point, and 0° at the boiling point.

Lambert's *Photometry* takes up three volumes (Nos. 31, 32, 33). Lambert was a foremost member of that brilliant band of talented men which made the eighteenth century a classical period in science. His versatility is remarkable, and as he was almost entirely self-taught and worked the fields of knowledge after his own sturdy fashion, he is both original and instructive, but at the same time diffuse. He was concerned mostly with general points of view and negligent in his experiments. His entire apparatus while constructing his *Photometry* (which is a pioneer-work in its branch) consisted of three little mirrors, two lenses, a pair of glass plates, and a prism. He persisted in using these instruments even in Berlin, where the best apparatus stood at his disposal, and his skill in the manipulation of his tools is remarkable. The treatise on *Photometry* is largely antiquated, yet the charm of its originality, its solid nucleus of truth, still render it a readable work. "Delivered to-day," says the editor, E. Anding, "it would, despite its diffuseness, specialisation, and repetitions, form an excellent lecture-course in photometric methods." Lambert's character and heart are highly lauded by his contemporaries, and it is said that his fine countenance gave Lavater the first suggestion and stimulus to his physiognomical studies.

An extremely important number is that devoted to the researches on the *Expansive Law of Gases* (No. 44), and containing the papers of Gay-Lussac, Dalton, Dulong, Petit, Rudberg, Magnus, and Regnault. This succession of researches, comprised within the modest compass of 200 pages, is intimately connected with the enunciation of the notion of absolute temperature, and constitutes by the vicissitudes of its development one of the most instructive chapters in the history of science.

Number 63 is devoted to the first researches in Electromagnetism and contains Oersted's brief account of his discovery of the deflexion of a magnetic needle by an electric current, as also an abstract of Seebeck's lectures on the Magnetism of the Galvanic Circuit.

The extraordinary work of Sadi Carnot, *Réflexions sur la puissance motrice du feu*, etc., forms No. 37. Carnot died at the early age of thirty-six (at the same age as Hertz), and his work, though containing the germs of much that was necessary to the formulation of the principle of the conservation of energy, lay almost

unnoticed for a quarter of a century. If we could interpret Carnot's ideas by the right intellectual environment we should be justified in denominating him the discoverer of the important principle known as the first law of thermodynamics. It is certain that his methods led to its discovery and that his work contains substantially the material now formulated in the second law. Carnot's results were known to Helmholtz, whose treatise on the Conservation of Force, with Helmholtz's own notes, edited in 1889, forms the first issue of Ostwald's *Classics*.

Finally, we have in Physics and Astronomy the Spectrum Analysis of Kirchhoff and Bunsen, Gauss's researches on Terrestrial Magnetism and on Forces Acting Inversely as the Square of the Distances, Bessel on the Length of the Second's Pendulum, Neumann on the Mathematical Theory of Induced Electric Currents, Kant on the Theory of the Heavens, Coulomb, Galvani, Hittorf and Seebeck on Electricity and Magnetism, Lavoisier and Laplace on Heat, and so on. In Botany and Physiology but few numbers have as yet appeared. They are essays by Saussure, Pasteur, Kölreuter, Sprengel, Knight, Weber, Ludwig, Becher, Rahn, and Ernst Brücke.

The department of Chemistry alone remains. As might be expected, it is richly represented. The *Dissertation on Fire and Water* (No. 58), by Carl Wilhelm Scheele, the Swedish chemist, written in 1777, remains to-day a marvel of simplicity. A person of common education may read the little book and repeat its experiments with the instruments and ideas which every-day life affords. Scheele, in Ostwald's opinion, possessed the distinctive qualifications of the chemist in their highest development, his experimental skill and powers of inference having never before or since been reached. No. 3 gives the treatises of Dalton and Wollaston on the Atomic Theory. The papers of Dalton are interesting as showing how with inexact analysis and experiments Dalton's thought yet compassed and enunciated so important a principle as the atomic hypothesis. We have here also the first table of atomic weights and the enunciation of Dalton's important theory of the constitution of bodies, and his law of constant and multiple proportions. The paper of Wollaston supplements Dalton's work, and gives experiments that for facility and cogency may be regarded to-day as the best experimental demonstrations of Dalton's laws. It is perhaps unknown to the majority of students that Wollaston was the first who attempted to draw up a more exact picture of the nature of chemical combination by the spatial disposition of atoms.

The speculative researches of Avogadro and Ampère on the foundations of the molecular theory are given in No. 8, the researches of Berthollet on the laws of affinity in No. 74, and the famous investigations of Berthollet's pupil, Gay-Lussac, on iodine in No. 4. Gay-Lussac's paper is accounted the most perfect and exhaustive original investigation of a single chemical element that exists. The discovery of a new element has never been exploited with such thoroughness as in this monograph of the great French chemist. The series also contains the treatises of

Meyer and Mendelejeff, and dissertations by Liebig, Bunsen, Pasteur, Berzelius, Davy, etc.

In the case of many of the older investigators, the editors of the Series have reproduced only what they deemed important. The Series is not, therefore, in all cases a *full* reprint of the scientific classics. It might have been desirable, further, to print the texts of the originals along with the German translations. Although probably not warranted from a commercial point of view, this step would have made the Series international in its character and usefulness. Altogether, we cannot close without words of high commendation for the undertaking, nor without expressing the hope that its range of usefulness will be extensive and its fruits beneficent. THOMAS J. MCCORMACK.

VORLESUNGEN UEBER GESCHICHTE DER MATHEMATIK. By *Moritz Cantor*. Leipsic: B. G. Teubner. 1894–1896. Price, Vol. I., 22 Marks; Vol. II., 24 Marks; Vol. III., Two Installments, 12 Marks.

It would be impossible to do justice to this monumental work within the brief limits of a book review, even if the task were not rendered supererogatory by the high standing of the work and the acknowledged authority of its author. Cantor's *Lectures on the History of Mathematics* are the work of a man who has unswervingly devoted a life-time to this single task, who thirty-three years ago was well known for his important contributions to this subject, and who can now in the second edition of the first volume of his great work point with pride to the impulse and awakened interest which his endeavors have aroused in the historical studies of his science. He has had many predecessors, each of whom has distinguished himself in certain branches and by certain excellences—Montucla who excelled in lucidity, elegance, and popularity; Libri who seems to have united in an eminent degree all the qualities necessary to the makeup of a writer of a universal mathematical history, but whose work extends only to the period preceding Galileo in Italy; Hankel, whose contributions to the history of early mathematics are marked by much acumen; and several others. Nevertheless, it may safely be said that profundity, accuracy, and extensiveness of treatment have never before in any history of mathematics been so thoroughly and intimately united as in the three volumes constituting these *Lectures* of Moritz Cantor. The first volume embraces the period from earliest antiquity to the year 1200 A. D. and is now in its second edition, thoroughly revised and brought down to date (1894). The second volume embraces the time from 1200 to 1668 A. D. The third and last volume will comprise the time from 1668 to 1759, concluding with the first epoch-making papers of Lagrange in the Proceedings of the Turin Academy. The first two installments only of this third volume have appeared (1894–1896), the third is still in preparation.

In the Introduction to Volume I., which contains 883 pages with a chart of ancient numerical characters, we have some brief philosophical considerations concerning the psychological origin of mathematical operations and the invention

of numerical signs. As to the theory that the first numerical words originally denoted not numbers but definite objects, Prof. Cantor remarks that philology has not succeeded in proving its position. Nor can he himself offer much to the solution of the problem. We are on sure ground, he says, only when we come to derivative numerical words. We have also some interesting remarks on the various systems of numbers, namely, the decimal, vigesimal, undecimal, sexigesimal systems, etc. The true history of mathematics, the author contends, begins only with the first written monuments and inscriptions which are presumably found in Egypt. 55 pages are devoted to the mathematics of Egypt, 31 to that of the Babylonians, 65 to that of the Indians, 29 to that of the Chinese, and 118 to that of the Arabs. The remaining three divisions of the first book are devoted to the mathematical achievements of the Greeks, which naturally take up the largest space, and to those of the Romans and of the early mediæval monasteries. The researches of the ancient nations are extremely interesting, not only from the point of view of mathematical history but equally so from that of philosophy and psychology. Their insight and errors are of extreme importance, and it is both profitable and fascinating to witness the primitive operations of the human mind as employed upon this its surest and most fundamental subject. Of the Greeks the most interesting chapters are those relating to Pythagoras and Archimedes. Dr. Cantor gave long ago, in his *Mathematische Beiträge zum Kulturleben der Völker*, 1863, a charming appreciation of the life and achievements of Pythagoras, only differing from the chapter on the great philosopher in the present work by being more popular and less exhaustive. In Archimedes we have the man who may be regarded as the incarnation of the mathematical genius of antiquity, and the chapter devoted to him shows at its best the precious heritage which he left to us. It is surprising to note to what a pitch the Indians advanced arithmetic and algebra, and also to follow the work of the Arabs. In fine, the entire first volume is a book which can be read and consulted by writers of average elementary mathematical attainments, and offers material from which all readers may draw profit and entertainment.

The year 1200 was an important one in the history of European Mathematics, and is fitly chosen as the beginning of the second volume. Christianity was then in possession of the art of arithmetic, as it had been recovered from its different ancient and Eastern sources. It was also in possession of the zero and of the the no less important principle of the positional value of figures. Algebra, as far as equations of the first and second degree, had been compassed, the geometry of Euclid, the astronomy of Ptolemy, the writings of Theodosius, and of Menelaus, existed in Latin translations, and appositely to the right time came the right men who were destined to achieve great things in mathematical science,—Leonardo of Pisa and Jordanus Nemorarius.

"Leonardo," says Cantor, "was a practised arithmetician and geometer, an ingenious algebraist, conversant with the application of algebra to geometry, as well as a creative genius of high rank in the theory of numbers." Jordanus

Nemorarius was a priest and member of a powerful order; he fell little short of Leonardo in point of mathematical ability, but by reason of his ecclesiastical position his influence was more powerful and decisive than that of the other who was a merchant. From these two great landmarks the second volume traces the history of mathematics through the early developments of algebra and geometry in England, France, Italy, and Germany, including Nicolaus of Cusa, Regiomontanus, Leonardo da Vinci, Luca Paciuolo, Michael Stifel, etc., down to the researches on cubic equations by Cardano and Tartaglia, where the first installment ends. The second installment is devoted to the advances made in cyclometry and trigonometry by Vieta, Van Roomen, etc., to the researches on equations of the fourth degree by Bombelli, etc., Kepler's and Pascal's investigations in geometry, the rise of mechanics, logarithms, continued fractions, the theory of numbers, analytical geometry, and lastly to the germs of the infinitesimal calculus in Kepler, Cavalieri, and most notably of all in Fermat. The volume concludes with the year 1668–1669, a momentous epoch in the history of mathematics, for at that time Gottfried Wilhelm Leibnitz was publishing at Leipsic his Doctor's dissertation, and Isaac Newton had just been elected to the chair of Mathematics in Cambridge University England.

With this epoch the second volume begins. The period which follows is of all that of most import for modern mathematics, and its utterances are associated with the most interest for professional readers. The first installment deals with the "geometrical character" of Leibnitz, with certain developments of commercial arithmetic, with the history of series as developed by Mercator, Brouncker, Gregory, Newton, Leibnitz, Halley, De Moivre, James Bernoulli, with continued fractions, the theory of curves, etc. We have also in this installment a chapter on Newton and Leibnitz's first discoveries in the domain of the infinitesimal calculus, chapters on Leibnitz and on the brothers Bernoulli both preceding and during their famous strife. The great controversy concerning the priority of invention of the differential calculus between the followers of Newton and Leibnitz,—a controversy which excited the mathematical world for more than twenty-five years, and which was really not definitively settled until the present century,—takes up a good part of the second and latest installment of Cantor's third volume. There is now, of course, little to be said upon the subject of this controversy, and Cantor does not claim to add much to its elucidation, except to point out an omission made in the copying of a letter by Leibnitz to Wallis of the word *hodie*, which might easily have led to certain suspicions in the English mind as to Leibnitz's fair dealings. His conclusion is that now that both great inquirers have received their just share of the credit owing to them for their discoveries, a careful and unprejudiced examination of the controversy unfortunately shows that the conduct of the matter reflected no little discredit upon *all* parties concerned. The last installment closes with the developments of the calculus, of Algebra, and of analytical and projective geometry down to the year 1726. The work of Euler and his period remains.

Cantor's history now comprises 2218 pages. The final installment, reaching to the year 1759, and which is yet to appear, will certainly not increase its bulk to much over 2600 pages, leaving the vast material from the date of Lagrange's first memoirs on to be elaborated by another hand. The history will thus hardly exceed in size some of its predecessors, but it will contain proportionately more material, from its being almost exclusively devoted to the solider scientific aspects of its subject and not so much to biographical and personal details, which served so greatly to swell the work of Montucla. In fine, it is far and away the concisest, yet most comprehensive and authoritative treatment of the subject that we have. As such, it is the indispensable adjunct of every mathematical worker and absolutely necessary in every mathematical library. T. J. McC.

PHYSIKALISCH-CHEMISCHE PROPAEDEUTIK. Unter besonderer Berücksichtigung der medicinischen Wissenschaften und mit historischen und biographischen Angaben. Von *Professor Dr. Med. et Phil. H. Griesbach.* Erste Hälfte. 272 Pages. Price, M. 6. Zweite Hälfte, I. Lieferung, 320 Pages. Price, M. 7. Leipsic: Wilhelm Engelmann. 1895 and 1896.

The present work is in the nature of an encyclopædic introduction to medicine, and deals with the specific chemical and physical facts, as well as methods, which enter into the foundations and structure of that science. The work is published in two parts, comprising three installments of some 300 pages each, and covers an unusually vast field. Its author is a man of scientific attainments and of wide and profound bibliographical knowledge. He has materially added to the attractiveness of the work by interweaving with his expositions a great mass of biographical and historical data. Each subject treated acquires thus a developmental form, well adapted to strengthening the memory of the student for the different subjects. Altogether, we have in the book an abridged history of science, and even of philosophy, the main subjects of which are also incidentally touched upon. Since the work presumes no special scientific or mathematical knowledge, it may be used with profit by every student, no matter what his profession or sphere of activity, the material it offers being such as should be known by every educated member of society. Further, on all the subjects coming within the designation of the "propædeutics of physics and chemistry" it constitutes a valuable reference book of the facts, and more especially of the literature, as also an etymological dictionary of scientific terms. That many dubious philosophical considerations should have slipped into a work which covers so vast a field and sounds the depths of so many sciences is natural and intelligible. This we shall see in the following review of the contents:

We have in Chapter I. a discussion of the character of science and logic; in Chapter II. a discussion of the character, method, and aim of physical science; Chapter III. treats of the origin of physical and chemical science and of scientific observation; in Chapter IV. space and time are treated. Here the author takes the

position that the questions why space is three-dimensional and time is one-dimensional, are problems that lie totally without the bounds of human comprehension. Even his own views on the subject are not confidently pronounced, for who, he says, would dare to assert he had found the solution of questions thus hovering at the boundary-line of human thought.

In Chapter V. we have a brief note on causality. In Chapters VI., VII., and VIII. we have a good presentation of the principles of mensuration and of metrical systems, of the graphic representation of natural phenomena, and of the measurement of space and time, all of which is accompanied by appropriate descriptions and illustrations of instruments and methods. One of the most important chapters is that on matter, energy, work, and force, into which considerable metaphysical speculation has been introduced. 'Apart from mind,' the author asserts, 'we 'find but one thing possessing real and absolute existence in the world, and this 'one thing we call substance. Substance comprises matter and energy, and when 'we speak of matter and energy we must be understood as making the tacit as-'sumption that both are simply integral parts of one and the same substance.'

We catch at once the author's metaphysical point of view. He says further: 'That which science calls matter is identical with but one of the component parts 'of the substance present in the physical cosmos. Further, it is practically impos-'sible to conceive of dynamic effects as not proceeding from some vehicle. Conse-'quently energy, as the component part of a substance, must itself be substantial, 'has the same right to be considered such as matter. Energy is not an independent 'substance, but, combined with and supplementing matter, it forms, together with 'the latter, the ultimate uncreatable and indestructible substance that constitutes 'the physical All.'

Heat, light, and electricity are sub-species of energy, and the author finds no philosophical impediment in saying that energy *possesses* a capacity to perform work. The development of the theory of energy has been made the basis of this work, and the philosophical interpretation of its significance is a point upon which the author apparently lays great stress. We have only to add that so deeply has the power of the chemical and molecular theories of physics impressed his mind that he actually proposes a molecular hypothesis of energy. Even Professor Ostwald, who has approached this conception very nearly, writes in a private letter to Dr. Griesbach that he sees at present no occasion for a molecular hypothesis of energy. It is certainly difficult to see what satisfaction the solution of a problem can give which simply refers its difficulties farther and farther back and associates them with less palpable and more tenuous particles. If such theories can satisfy the mind in the long run, it will not be long before we shall be conceiving of motion as a substance.

Chapter XI. treats of the measurement of velocity; Chapters XII. and XIII. of centrifugal forces and their practical applications, of friction and obstacles to motion; Chapters XIV., XV., XVI., and XVII. treat of the divisibility and constitu-

tion of ponderable matter, of the important question of the constitution of the ether, of the history of atomistic theories and of organic and inorganic matter. In the following chapters, so far as the second installment, we have discussions of the porosity of matter, with demonstrations and suggestions of its significance in applied science. Atmospheric pressure is treated, barometers, and manometers fully described, and finally, there is a long and important chapter on aggregate states of matter. The biological and physiological chapters in the first two installments contain a wealth of material, brought down to date. The pathogenic properties of organised matter are treated of here, the conditions of fermentation and of the production of disease by bacteria, with good studies of typical forms of micro-organisms. The bibliography is particularly full and valuable.

The third installment, which has not yet reached our hands, will deal mainly with the science of energetics, including heat, gravitation, radiant and chemical energy, discussing the sources of energy, its laws, the foundations of modern chemistry, and not omitting other branches of physics which are of importance in the propædeutical studies which the author has in view. μκρ.

ESSAIS SUR LA PHILOSOPHIE DES SCIENCES. Analyse.—Mécanique. Par C. De Freycinet. Paris: Gauthier-Villars et fils. 1896. Pages, 336.

M. Freycinet seeks to answer such questions as, What is the exact nature of the notions of infinity and infinitesimal quantities whereon the higher analysis rests? Wherein does the "invention of Leibnitz" differ from the common algebra? What share of the contents of mechanical principles is to be assigned to reasoning and what to experience? What assures the conservation of force and energy? May we predict a gradual slackening of the causes that agitate matter? And so forth.

The notions of Analysis, M. Freycinet contends, are derived directly from the notions of space and time, which for him are necessary, infinite, continuous, and homogeneous. His speculations on this topic are essentially based upon the reflexions of Pascal who, he says, would certainly have invented the Differential Calculus had he not been early called away from science by his excessive religiosity. Infinity is immanent in nature and inherent in mind, escaping intimate comprehension, yet serving accurately our purposes,—a necessary attribute of the world of sense and intellect: and hence its power. The parallelism of mind and nature, in fact, runs all through M. Freycinet's book, and furnishes him with a satisfactory key to many metaphysical problems. So here, after an examination of the Calculus and of its applicability to Physics, he finds "that the Infinitesimal Analysis is alike admirably adapted to the phenomena of nature and to the conceptions of human reason,—apparently forming a bond of union between the intellect and the outer world, which is the highest commendation one can bestow upon it." And the same consideration is applied to the notions of Mechanics, where it is said that "the human mind and nature form integral parts of the same system, by virtue of which the one is richly equipped for the comprehension of the other;" and he illustrates his idea by the example of the Apollonian discovery of conic sections, centuries before their employment as a model of the planetary system. Generally Mr. Freycinet's reflexions upon the subject of limits and the infinitesimal method are lucid and unobjectionable, and from their simplicity may be re-

commended to elementary students. He finds the two ideas of limits and infinitesimals to be conjoint, correlative notions, not at all illogical, and sees the difference of common algebra and the infinitesimal method in the sameness, the simple more-or-less-ness, of the quantities dealt with by the former, and the non-identity, not excluding a sort of homogeneity, of the variables and limits of the latter.

In the chapters on Mechanics, we have numerous elucidative discussions, at times not unmixed with metaphysics. On the ground that the slightest impulse can impart motion to the largest mass, we are led to the statement that "resistance is never *in* the body but always *without* the body,"—a proposition full of light and truth, but entirely depending on the definition of "resistance," and when true only equivalent to its premise. After an examination of the circle-argument involved in the description of mass in terms of quantity of matter, mass is defined as "the expression of relative mobility."

One ingenious point is the enunciation of the idea of *dynamic capacity*,—an analogue of calorific capacity, or of the idea of specific heat. We may say, according to the author, that the same volumes of water, lead, mercury, etc., *absorb* different quantities of force or "impulsion," just as they do different quantities of heat. And as we construct scales of specific heats, so we could construct dynamic scales of bodies, which would give what is commonly called their "quantity of matter" or mass. We see here the form Physics might have taken on, had it been possible to start from heat instead of motions of masses. The idea, at least in its order, is not new.[1]

M. Freycinet insists clearly and repeatedly on the separation of experience from reason in the contents of Mechanics, and also on many other sound fundamental doctrines. We have not time to enter into the physical metaphysics of the latter chapters of M. Freycinet's book; we wish merely to indicate the scope and general aim of the work. M. Freycinet is a distinguished French engineer, a member of the National Institute, and already well known as a writer upon the philosophical aspects of scientific questions. He has always applied himself by predilection to the questions involved in the epistemological foundations of the Calculus and mechanics, and his present work is a continuation of former investigations in this domain. One is constrained to admire the conciseness and directness of his expositions, as also the apt and simple style in which they are conveyed. Altogether we have a very readable book, combining commendable internal and external excellences.

T. J. McC.

N. B.—Reviews of works by Dr. Jodl, Dr. Eucken, Dr. Mach, Dr. Ratto, and others have been crowded out of the present *Monist*, as have also the "Contents of Periodicals."

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[1] See Mach, *Ueber die Erhaltung der Arbeit*, Prague, 1872; *Popular Scientific Lectures*, Chicago, 1894, pp. 168-171.